

Problem Set 1: Review of Mathematics; Aspects of the Business Cycle

Questions 1 to 5 are intended to help you remember and practice some of the mathematical concepts you may have encountered previously. Practicing these will be useful for the course which often illustrates some concepts mathematically. We will use these techniques and concepts at various points during the course. Answer the following questions. Reference materials for these questions can be found in Lecture 2 and the Math Review Notes.

1. Graphing question. Consider the following equation:

$$y = a + bx$$

- a) Suppose that $a = 10, b = -2$. Sketch this function.
- b) What would happen if b fell to -1 ?
- c) What would happen if a increased to 20 ?
- d) Suppose that $a = -2, b = 2$. Sketch this function.
- e) What happens if b increases to 4 ?
- f) What happens if a falls to -5 ?

2. Consider the following equations. They are written as y as a function of x , i.e. $y = f(x)$. For each of the equations, calculate the slope, $f'(x)$, of the graph of y (on the vertical axis) against x (on the horizontal axis).

- a) $y = 2x + 10$
- b) $y = 5 - x$
- c) $4y = 2x - 5$
- d) $2y = -(4 + 8x)$

3. Consider all the functions in question 2(a) -(d). They are written as y as a function of x , i.e. $y = f(x)$. Write out the inverse function, that is to say, write x as a function of y , i.e. $x = g(y)$.

4. For each of the equations in the parts below, figure out how much y changes if x changes by 1, i.e. calculate $\frac{\Delta y}{\Delta x}$, when $\Delta x = 1$.

a) $y = f(x) = 10x + 5$

b) $y = f(x) = 0.75x + 0.25z - 15z^2$.

c) $y = f(x) = \frac{150+5g-2w+20x}{10} + 200$

d) $y = f(x) = \frac{150+5g-2w+x}{1-c} + 200$, where c is a parameter, and g, w and x are all variables.

5. Consider the following equation:

$$Y = \frac{c_0 - c_1T + I + G + X - M_0}{1 - c_1} - \frac{b}{1 - c_1}r$$

where c_0, I, G, X, M_0, r are all variables, and c_1 and b are parameters.

a) What is the resultant change in Y from a change in G , i.e. $\frac{\Delta Y}{\Delta G}$?

b) What is the resultant change in Y from a change in T , i.e. $\frac{\Delta Y}{\Delta T}$?

c) [A little trickier:] What is the resultant change in r from a change in Y , i.e. $\frac{\Delta r}{\Delta Y}$?

The following questions relate to the business cycle. For this, you will need to use Excel and be able to access the dataset for the United Kingdom, available at:

<http://facstaff.uww.edu/ahmady/courses/econ736/data/ps1data.xls>

The first worksheet contains the full dataset. The subsequent worksheets labeled ‘Growth Rate’, ‘Inflation’, ‘NIPA’, and ‘Correlations’ are used in the questions that follow.

6. GDP.

Consider that for a variable, x_t , the growth rate of x is calculated between periods $t - 1$ and t as follows:

$$\text{Growth Rate of } x = \% \Delta x = \frac{\Delta x_t}{x_{t-1}} * 100 = \frac{x_t - x_{t-1}}{x_{t-1}} * 100$$

Click on the worksheet titled “Growth Rate”. This sheet has the Nominal GDP and the GDP Deflator copied over from the full dataset.

- a. Calculate Real GDP using the nominal GDP and GDP deflator data in column D.
- b. Using the formula above, calculate the Annualized Economic Growth Rate (i.e. the percentage change in real GDP between one quarter and four quarters previously) in column E. [Note: You will not be able to calculate growth rates from 1973Q1 - 1973Q4, since you would need data from 1972Q1-Q4].
- c. What was the historical average growth rate for the UK (- you may find the “=AVERAGE()” worksheet function useful here!)? How was the UK growing at the beginning of 2006 relative to its historic average?
- d. Consider that a popular working definition for a recession is at least two consecutive quarters (i.e. 6 months) of negative economic growth. Based on your calculations, when did the UK face a recession(s)? [Hint: you are looking for *at least 2* consecutive quarters of negative economic growth. You may find it useful to use the “=IF(,)” formula in column F to check when the corresponding number in column E is negative for each period.]
 - e. [*] Plot the economic growth rate for the UK in a chart. The UK experienced a mild recession in 2001. Is this recession reflected in the graph that you just plotted? Explain why or why not!
 - f. [*] In column H, calculate the natural log of the real GDP you calculated in column D. Then in column I calculate the difference in the log of the real GDP for a particular period and the four quarters previous to it; multiply the result by 100. As in part b, you will not be able to calculate values for 1973Q1-Q4 since you do not have data for 1972. What have you just calculated? How does what you calculate compare to what you calculated in column E? If your numbers are different, explain why.

7. Inflation.

Turn to the worksheet labeled “Inflation”. This worksheet has the year, and data from the GDP Deflator and CPI copied in. It also has a column called ‘Index’ which shows the respective quarter of the year.

- a. Plot the GDP Deflator and the CPI against each other. How do they compare?
- b. Calculate the average annualized price index, and write the results in column K for the GDP Deflator and column L for the CPI. [Hint: One way to do this is to work out the average for the current period and the previous three periods. Do this for both the GDP Deflator and the CPI in the respective columns. If you have done this correctly, then every fourth row, i.e. in every row which is Q4, it will contain the average value for the price index for that year. Then all you have to do is highlight the data from A1:F136 and use the autofilter option: go to the menu (in Excel 1997-2003 versions): Data>Filter>Autofilter. You should see little pull down menu’s for each column. Go to the index column, and click on the pull down menu. Select ‘4’ which will only show the data relating to the fourth quarter. Now highlight and copy the numbers in columns E and F to a different part of the worksheet. Turn the autofilter off by going back to Data>Filter>Autofilter and selecting it again (to turn it off). Then copy the results to columns K and L. All this can be similarly done in Excel 2007.]
- c. Using your answer to part (b), what year is the base year for the GDP deflator and CPI series?
- d. What is the correlation between the GDP Deflator and the CPI (using the quarterly data)? [Hint: You may find the “=CORREL()” worksheet function useful here].
- e. What is a reason for the difference in the numbers between the GDP Deflator and the CPI? Can you think of an example of a good that appears in the CPI and not in the GDP deflator, and vice versa?
- f. In columns G and H, calculate inflation using both the GDP Deflator and the CPI from

columns C and D respectively. What were the historical average inflation rates based on the GDP deflator and the CPI?

8. National Income and Product Accounts (NIPA)

Turn to the worksheet labeled NIPA. This worksheet has Nominal GDP, the CPI and nominal data on consumption, investment, government purchases and net exports from the original data set.

a. Copy and paste in your values for Real GDP that you calculated earlier into column C. [Hint: Make sure you use the "Paste Special" command and then paste in the actual values as opposed to the formulas themselves.]

b. Using the CPI as the price level, calculate the values of real consumption, investment government purchases and net exports in columns I - L.

c. Calculate the correlations between real GDP and each of these variables (C, I, G and NX) both for the nominal variables and the real variables. How would you classify how C,I,G and NX behaves with regards to the business cycle (i.e. are they procyclical, countercyclical or acyclical)? For each, suggest an explanation as to why they move in the way that they do. [Recall: the closer the correlation is to zero, the more it exhibits acyclical behavior; the closer it is to +1 (-1), the more procyclical (countercyclical) it is].

d. In columns M - P, calculate the share of each of the real variables (i.e. C, I, G and NX) as a fraction of real GDP. Historically, what fraction of real GDP is made up of consumption, investment, government purchases and net exports? Which is the biggest?

e. In columns Q-T, calculate the growth rate of C, I, G and NX. Out of consumption, investment and government purchases, which is the most volatile? [Hint: you may find the variance worksheet function, VAR() helpful here.]

9. Comparison with the Business Cycle.

Turn to the worksheet labeled "Correlations". This worksheet has copied the data on

the GDP Deflator, the CPI, unemployment, the unemployment rate, nominal wages, money market interest rates, and the money supply from the original data set.

a. From your answer to question 6(a), copy the values of Real GDP you calculated into column C. Also, copy your values for the GDP Deflator based inflation and CPI based inflation into columns F and H respectively. As before, make sure you use the "Paste Special" command and then paste in the actual values as opposed to the formulas themselves. In column D, calculate the quarterly economic growth rate (as opposed to the actual annual economic growth rate as you did in question 6).

b. Suppose that real wage is determined as:

$$\text{Real Wages} = 100 * \frac{\text{Nominal Wages}}{\text{Price Level}}$$

Calculate real wages for the UK in column L using the CPI as the measure of the price level.

c. Recall that velocity represents the speed at which money circulates in an economy for the purpose of transactions. In column N, calculate velocity using the formula for velocity below:

$$\text{Velocity} = \frac{\text{Nominal GDP}}{\text{Money}}$$

d. [*] Determine the type of cyclicity that each of the following series exhibits (i.e. are they procyclical, countercyclical, or acyclical): GDP Deflator, GDP Deflator based inflation, CPI, CPI based inflation, unemployment, the unemployment rate, nominal wages, real wages, money market interest rates, money supply and velocity. For each series, suggest an explanation for the type of cyclicity exhibited. [Hint: for variables that are in levels (as opposed to percentage changes), it makes sense to look at the correlation between real GDP and that variable; for variables that are already a percentage change, you probably want to look at its correlation with the percentage change in real GDP, i.e. the economic growth rate.]