

Business Conditions Analysis

ECON 736

Professor Yamin Ahmad

- Review of Mathematics



In This Lecture...

Review of Mathematics

- Graphs
- Slopes

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Graphing Data

- Economists use three types of graphs to reveal relationships between variables. They are:
 - Time-series graphs
 - Cross-section graphs
 - Scatter diagrams

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Graphing Data

Time-Series Graphs

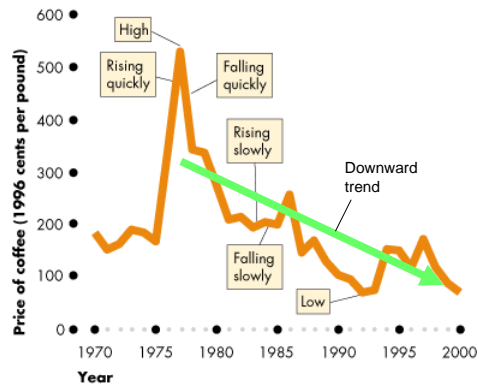
- A **time-series graph** measures time (for example, months or years) along the x-axis and the variable or variables in which we are interested along the y-axis.
- The time-series graph on the next slide shows the price of coffee between 1970 and 2000.
- The graph shows the level of the price, how it has changed over time, when change was rapid or slow, and whether there was any trend.

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Graphing Data



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Graphing Data

Cross-Section Graphs

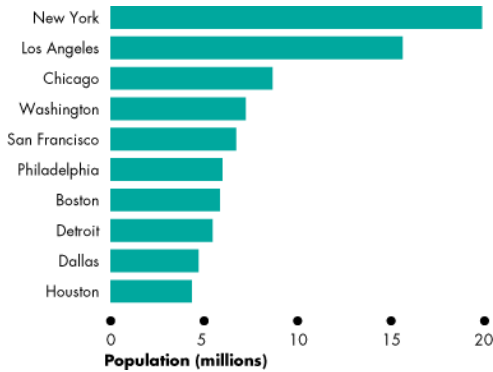
- A **cross-section graph** shows the values of a variable for different groups in a population at a point in time.
- The cross-section graph on the next slide enables you to compare the number of people who live in 10 metropolitan areas in the United States.

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Graphing Data



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Graphing Data

Scatter Diagrams

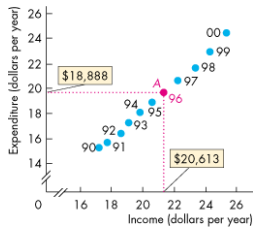
- A **scatter diagram** plots the value of one variable on the x-axis and the value of another variable on the y-axis.
- A scatter diagram can make clear the relationship between two variables.
- The three scatter diagrams on the next slide show examples of variables that move in the same direction, in opposite directions, and in no particular relationship to each other.

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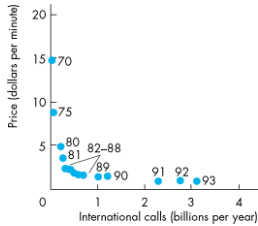
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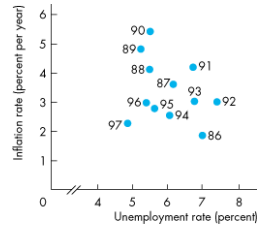
Graphing Data



(a) Expenditure and income



(b) International phone calls and prices



(c) Unemployment and inflation

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Graphs Used in Economic Models

- Graphs are used in economic models to show the relationship between variables.
- The patterns to look for in graphs are the four cases in which:
 - Variables move in the same direction
 - Variables move in opposite directions
 - Variables have a maximum or a minimum (extremals)
 - Variables are unrelated

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Graphs Used in Economic Models

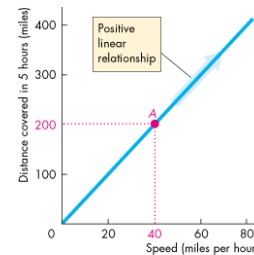
Variables That Move in the Same Direction

- A relationship between two variables that move in the same direction is called a **positive relationship** or a **direct relationship**.
- A line that slopes upward shows a positive relationship.
- A relationship shown by a straight line is called a **linear relationship**.
- The three graphs on the next slide show positive relationships.

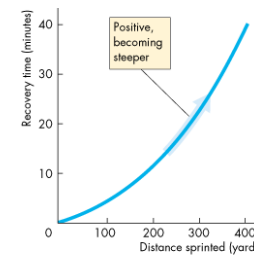
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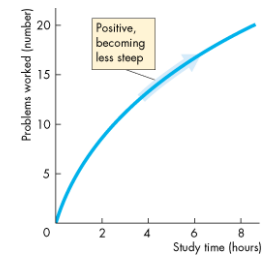
Graphs Used in Economic Models



(a) Positive linear relationship



(b) Positive, becoming steeper



(c) Positive, becoming less steep

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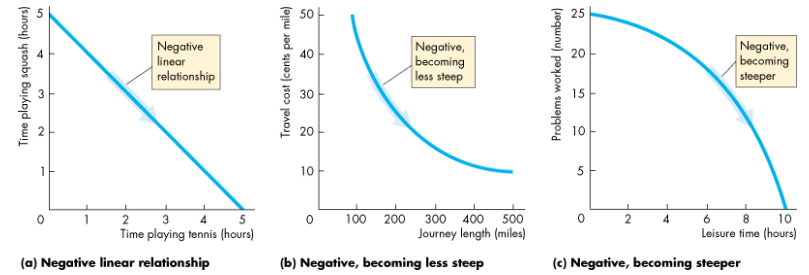
Graphs Used in Economic Models

Variables That Move in Opposite Directions

- A relationship between two variables that move in opposite directions is called a **negative relationship** or an **inverse relationship**.
- A line that slopes downward shows a negative relationship.
- The three graphs on the next slide show negative relationships.



Graphs Used in Economic Models



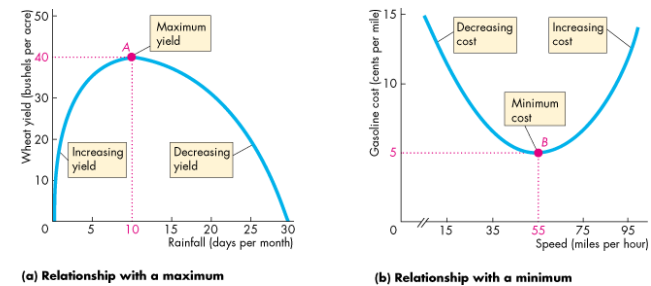
Graphs Used in Economic Models

Extremals: Variables That Have a Maximum or a Minimum

- The two graphs on the next slide show relationships that have a maximum and a minimum.
- These relationships are positive over part of their range and negative over the other part.



Graphs Used in Economic Models





Graphs Used in Economic Models

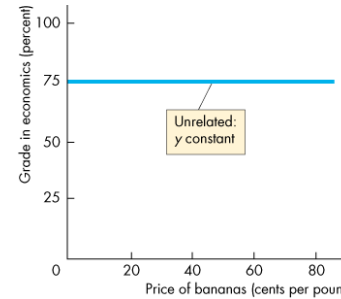
Variables That are Unrelated

- Sometimes we want to emphasize that two variables are unrelated.
- The two graphs on the next slide show examples of variables that are unrelated.

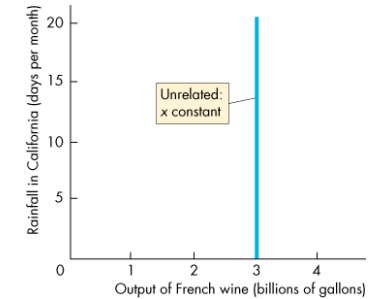
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Graphs Used in Economic Models



(a) Unrelated: y constant



(b) Unrelated: x constant

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The Slope of a Relationship

- The **slope** of a relationship is the change in the value of the variable measured on the y -axis divided by the change in the value of the variable measured on the x -axis.
- We use the Greek letter Δ (capital delta) to represent “change in.”
- So Δy means the change in the value of the variable measured on the y -axis and Δx means the change in the value of the variable measured on the x -axis.
- The slope of the relationship is $\Delta y/\Delta x$.

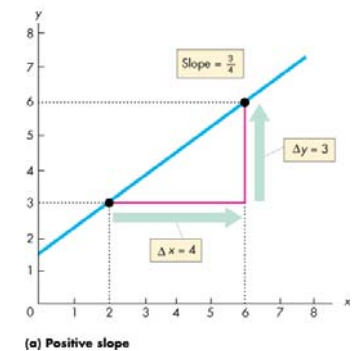
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The Slope of a Relationship

The Slope of a Straight Line

- The slope of a straight line is constant.
- Graphically, the slope is calculated as the “rise” over the “run.”
- The slope is positive if the line is upward sloping.



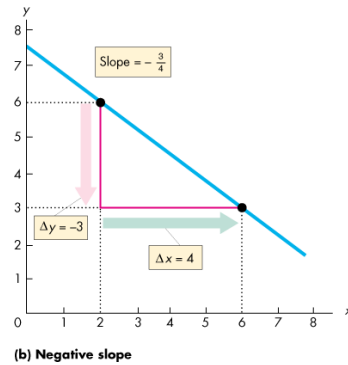
(a) Positive slope

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The Slope of a Relationship

- The slope is negative if the line is downward sloping.



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The Slope of a Relationship

- The Slope of a Curved Line
 - The slope of a curved line at a point varies depending on where along the curve it is calculated.
 - We can calculate the slope of a curved line either at a point or across an arc.

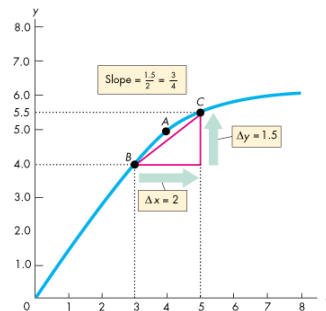
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The Slope of a Relationship

Slope Across an Arc

- The average slope of a curved line across an arc is equal to the slope of a straight line that joins the endpoints of the arc.
- Here, we calculate the average slope of the curve along the arc *BC*.



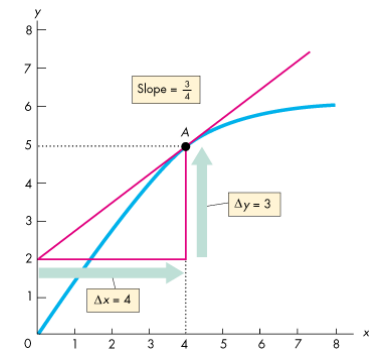
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The Slope of a Relationship

Slope at a Point

- The slope of a curved line at a point is equal to the slope of a straight line that is the tangent to that point.
- Here, we calculate the slope of the curve at point *A*.

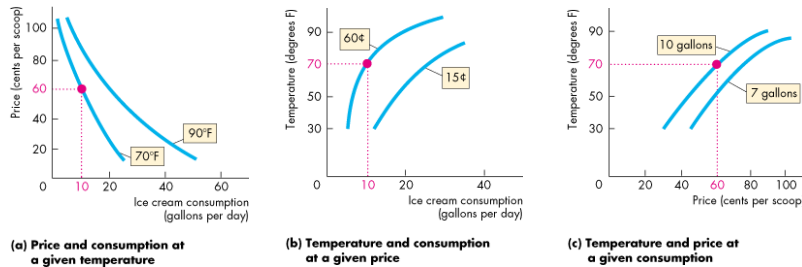


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Graphing Relationships Among More Than Two Variables

- When a relationship involves more than two variables, we can plot the relationship between two of the variables by holding other variables constant—by using *ceteris paribus*.
- Here we plot the relationships among three variables.



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Mathematics: What You Should Be Able To Do

- Graphs are a way to do math. But sometimes it is too hard to represent models with graphs. Then we use math.
- This moves us closer to what professional economists do:-
 - Algebra: Manipulate equations and solve for endogenous variables in terms of exogenous variables
 - Calculate the inverse of functions
 - Calculate the effects of changes in variables on other variables (- essentially working out slopes!)
 - Calculate percentage changes (- useful for calculating growth rates)
 - Examine correlations between variables

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More on Calculating Slopes

Slope of a Linear Function

- Consider a linear function:
- $$y = f(x) = mx + c$$
- Suppose that the pairs (x_0, y_0) and (x_1, y_1) both represent arbitrary points on the line. Then the ratio:

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$

is called the slope.

- c is called the intercept
- Question: What is the slope of a line that goes through the points $(4,6)$ and $(0,7)$?

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Solution...

- Take the points $(4,6)$ and $(0,7)$
- If we plug into the formula on the previous page:

$$\begin{aligned} \frac{\Delta y}{\Delta x} = m &= \frac{y_1 - y_0}{x_1 - x_0} \\ &= \frac{7 - 6}{0 - 4} = \frac{1}{(-4)} = -\frac{1}{4} \end{aligned}$$

- Hence the slope of a line that goes through the points $(4,6)$ and $(0,7)$ is $-1/4$.

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Another Practice Problem... (Solution next page)

Centigrade and Fahrenheit

- Let C denote the temperature in degrees Centigrade and let F denote the temperature in degrees Fahrenheit.
- We know that 0° Centigrade is 32° Fahrenheit is the freezing temperature of water, and that 100° Centigrade or 212° Fahrenheit is the boiling point for water.
- What is the equation that relates degrees Fahrenheit to degrees Centigrade? What is the inverse function, i.e. the equation that relates degrees Centigrade to degrees Fahrenheit?

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Practice Problem... Answer

Centigrade and Fahrenheit

- To find the equation of the line through points (0,32) and (100,212), we first find the slope:

$$\text{slope, } m = \frac{\Delta F}{\Delta C} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

- This means an increase of 1° Centigrade corresponds to an increase of 9/5° Fahrenheit. Hence we can use the slope, 9/5 and the point (0,32) to express the linear relationship:

$$\frac{F - 32}{C - 0} = \frac{9}{5} \quad \text{or} \quad F = \frac{9}{5}C + 32$$

- Or equivalently (re-arranging for $C=f(F)$)

$$9C = 5(F - 32) \quad \text{i.e.} \quad C = \frac{5}{9}(F - 32)$$

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Application to Economics...

Recall From Lecture 1...

- Models embody assumptions about individual behavior, market structure and what is exogenous (including policy regime).
- Solution gives us the endogenous variables in terms of the exogenous factors.

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Mathematical functions

- We use functional notation when we want to express the idea that one variable is determined by other variables.
 - For example, supply of pizzas is a function of the price of pizzas and the price of materials (price of inputs):

$$Q^s = S(P, P_m)$$
- In this example, the quantity supplied of pizza is the “endogenous” variable in the pizza supply model.
 - The price of pizza and the price of materials are “exogenous” for the pizza maker. She cannot influence those prices (assuming that she is not a monopoly seller of pizza!)

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More on Endogenous and Exogenous variables

- Variables that are exogenous in some models might be endogenous in other models.
 - For example, in many macro models, we might take the share of income earned by females vs. males as exogenous.
 - But some economic models are designed exactly to explain those shares.
- In some cases, a variable is exogenous in the building block of a more general model, but endogenous in the general model.
 - Price of pizza is exogenous for the pizza supplier, but determined within our model of the pizza market.

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The Pizza market

- We can take the equations for supply of pizza, demand for pizza, and market equilibrium:

$$Q^s = S(P, P_m)$$

$$Q^d = D(P, Y)$$

$$Q^s = Q^d$$
- P and P_m are exogenous for the pizza supplier. P and Y are exogenous for the pizza demander.
- These three equations together determine Q^s , Q^d , and P endogenously. The exogenous variables for the pizza market are P_m and Y.

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Macroeconomic example

- We will model aggregate consumption as depending on “disposable” income:

$$C = C(Y - T)$$
 - For consumers in this model, income and taxes are exogenous.
 - But aggregate income will be determined in our macro model.
- In most of our macro models, aggregate taxes will be exogenous, but sometimes they will be endogenous.
- In most of our models of consumer’s income is exogenous, but sometimes it is endogenous. Income depends on how many hours we work, for example.

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Algebra

- Review how to solve endogenous variables.
- **Task:** Solve for P and Q , in terms of P_m and Y :

$$Q = a + bP - cP_m \quad Q = d - eP + fY$$

- $a, b, c, d, e,$ and f are “parameters”. For example, e tells us how much demand falls when the price rises.



Solution

- Here is the solution:

$$Q = \frac{bd + ae}{b + e} - \frac{ce}{b + e} P_m + \frac{bf}{b + e} Y$$

$$P = \frac{d - a}{b + e} + \frac{c}{b + e} P_m + \frac{f}{b + e} Y$$

- A lot of information is embedded in these solutions:
 - e.g. amongst other things, we can see here how the sensitivity of supply to output prices, b , or the sensitivity of demand to income, f , influences equilibrium prices and quantity.