

Workshop III: Solutions

For each of the questions below, attempt them yourself and then discuss the questions within your individuals groups. I will ask groups to come to the board and show their solutions.

Numerical Problems

1. Desired consumption and investment are:

$$C^d = 4000 - 4000r + 0.2Y$$

$$I^d = 2400 - 4000r$$

As usual, Y is output and r is the real interest rate. Government purchases, G , are 2000.

- a. Find an equation relating desired national saving, S^d , to r and Y .

Answer:

$$\begin{aligned} S^d &= Y - C - G \\ &= Y - (4000 - 4000r + 0.2Y) - 2000 \\ &= -6000 + 4000r + 0.8Y \end{aligned}$$

- b. What value of the real interest rate clears the goods market when $Y = 10000$? Use both forms of the goods market equilibrium condition. What value of the real interest rate clears the goods market when $Y = 10200$? Graph the IS curve.

Answer:

Method 1:

$$\begin{aligned} Y &= C + I + G \\ &= 4000 - 4000r + 0.2Y + 2400 - 4000r + 2000 \\ &= 8400 - 8000r + 0.2Y \\ \Rightarrow Y &= 10500 - 10000r \end{aligned}$$

Method 2:

$$\begin{aligned} I &= S \Rightarrow \\ 2400 - 4000r &= -6000 + 4000r + 0.8Y \\ \Rightarrow 8400 - 8000r &= 0.8Y \\ \Rightarrow Y &= 10500 - 10000r \end{aligned}$$

Regardless of the method, we can now plug in the value of Y and solve for r as follows:

$$IS : Y = 10500 - 10000r$$

$$\Rightarrow r = \frac{10500}{10000} - \frac{1}{10000} Y$$

$$Y = 10000 \Rightarrow r = 1.05 - 1 = 0.05$$

$$Y = 10200 \Rightarrow r = 1.05 - 1.02 = 0.03$$

Thus for a higher level of output, real interest rates are lower. Graph would show a downwards sloping curve.

- c. Government purchases rise to 2400. How does this increase change the equation for national saving in Part (a)? What value of the real interest rate clears the goods market when $Y = 10000$? Use both forms of the goods market equilibrium condition. How is the IS curve affected by an increase in G ?

Answer: $\Delta G = 400$

$$S^d = -6400 + 4000r + 0.8Y$$

$$\therefore I = S \Rightarrow 2400 - 4000r = -6400 + 4000r + 0.8Y$$

$$\Rightarrow 8800 - 8000r = 0.8Y$$

$$\Rightarrow 11000 - 10000r = Y^*$$

$$\therefore Y = 10000 \Rightarrow r = 10\%$$

We get the same thing with the other approach of adding up expenditures. Hence an increase in G leads the IS curve to shift up/right.

2. In a particular economy the real money demand function is:

$$\frac{M^d}{P} = 300 + 0.1Y - 10000i$$

Assume that $M=6000$, $P=2$ and $\pi^e = 0.02$.

- a. What is the real interest rate, r , that clears the asset market when $Y = 8000$? When $Y = 9000$? Graph the LM curve.

Answer: Setting demand equal to supply for real money balances gives us:

$$\frac{6000}{2} = 300 + 0.1Y - 10000(r + 0.02)$$

$$\Rightarrow 3000 = 300 + 0.1Y - 10000r - 200$$

$$\therefore 10000r = -2900 + 0.1Y$$

$$\therefore LM : r = -0.29 + 0.00001Y$$

$$Y = 8000 \Rightarrow r = -0.29 + 0.08 = -0.21$$

$$Y = 9000 \Rightarrow r = -0.29 + 0.09 = -0.2$$

Thus the LM curve slopes upwards in (r, Y) space.

- b. Repeat part (a) for $M=6600$. How does the LM curve compare with the LM curve in part (a)?

Answer: $M=6600$ implies that:

$$\frac{6600}{2} = 300 + 0.1Y - 10000r - 200$$

$$\Rightarrow 10000r = -3200 + 0.1Y$$

$$\Rightarrow r = -0.32 + 0.00001Y$$

Thus an increase in the money supply shifts the LM curve downwards.

- c. Use $M = 6000$ again and repeat part (a) for $\pi^e = 0.03$. Compare the LM curve in this case with the one in part (a).

Answer: As in part (b), just re-derive the LM curve:

$$\frac{6000}{2} = 300 + 0.1Y - 10000(r + 0.03)$$

$$\Rightarrow 3000 = 0.1Y - 10000r$$

$$\therefore r = -0.3 + 0.00001Y$$

Thus an increase in inflationary expectations shifts the LM curve downwards.

3. An economy has full-employment output of 1000. Desired consumption and desired investment are:

$$C^d = 200 + 0.8(Y - T) - 500r$$

$$I^d = 200 - 500r$$

Government purchases are 196, and taxes are

$$T = 20 + 0.25Y$$

Money demand is:

$$\frac{M^d}{P} = 0.5Y - 250(r + \pi^e)$$

where the expected rate of inflation, $\pi^e = 0.1$. The nominal supply of money, $M = 9890$.

- a. What are the general equilibrium values of the real interest rate, price level, consumption and investment?

Answer: In general equilibrium, we need to solve for the goods and money market equilibrium and derive the price level. This can be done as follows:

Good's Market equilibrium:

$$Y = 200 + 0.8Y - 0.8(20 + 0.25Y) - 500r + 200 - 500r + 196$$

$$\Rightarrow Y = 580 + 0.6Y - 1000r$$

$$\therefore IS : Y = 1450 - 2500r \quad (1)$$

$$\therefore 2500r = 1450 - Y$$

$$\Rightarrow r = 0.58 - 0.0004Y$$

$$Y^* = 1000 \Rightarrow r = 0.18$$

$$\Rightarrow C^* = 694; I^* = 110; G = 196 (\Rightarrow C^* + I^* + G = 1000)$$

Money Market Equilibrium:

$$\frac{9890}{P} = 0.5Y - 250(r + \pi^e)$$

$$\Rightarrow \frac{9890}{P} = 500 - 250(0.18 + 0.1)$$

$$\Rightarrow P = \frac{9890}{430} = 23$$

- b. Suppose that government purchases are increased to $G=216$. What are the new general equilibrium values of the real interest rate, the price level, consumption and investment?

Answer: $\Delta G = 20$

$$\Rightarrow Y = 600 + 0.6Y - 1000r$$

$$\Rightarrow (\text{IS}') Y = 1500 - 2500r$$

$$\therefore r = 0.6 - 0.0004Y$$

So $Y = 1000 \Rightarrow r^* = 0.2$, which means interest rates have gone up (i.e. for the same Y , the IS curve is higher). To verify the values of C , I etc,

$$\left. \begin{array}{l} C^* = 684 \\ I^* = 100 \\ G = 216 \end{array} \right\} Y^* = 1000$$

Money Market equilibrium is thus given by:

$$\frac{9890}{P} = 500 - 250(0.2 + 0.1)$$

$$\therefore P = \frac{9890}{425} = 23.27$$

So prices are higher as well.