

# Business Cycles

## Econ 402

### Professor Yamin Ahmad

#### Lecture 2:

- Understanding and Interpreting Graphs in Economics



## Some Tools of Analysis

- Graphs:
  - Movements
  - Shifts of Curves
- Extremals: Maxima and Minima

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## Graphing Data

- Economists use three types of graphs to reveal relationships between variables. They are:
  - Time-series graphs ( - mostly in macroeconomics)
  - Cross-section graphs ( - mostly in microeconomics)
  - Scatter diagrams ( - in both, and in particular, econometrics)

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## Graphing Data

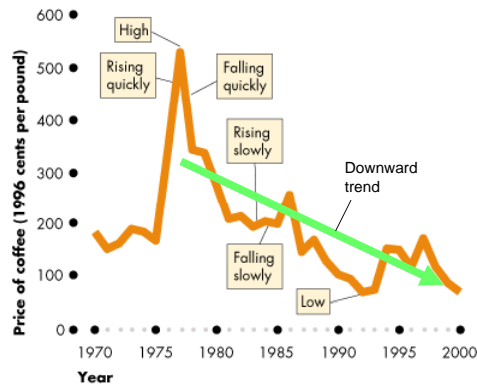
### Time-Series Graphs

- A **time-series graph** measures time (for example, months or years) along the x-axis and the variable or variables in which we are interested along the y-axis.
- The time-series graph on the next slide shows the price of coffee between 1970 and 2000.
- The graph shows the level of the price, how it has changed over time, when change was rapid or slow, and whether there was any trend.

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## Graphing Data



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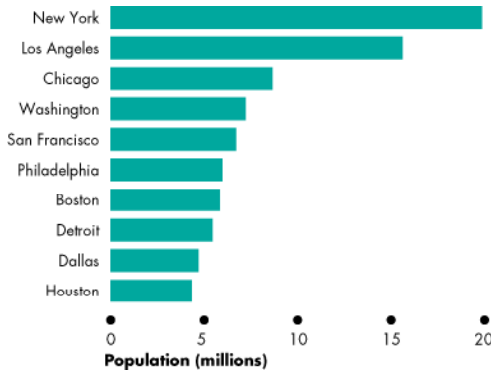
## Graphing Data

### Cross-Section Graphs

- A **cross-section graph** shows the values of a variable for different groups in a population at a point in time.
- The cross-section graph on the next slide enables you to compare the number of people who live in 10 metropolitan areas in the United States.

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## Graphing Data



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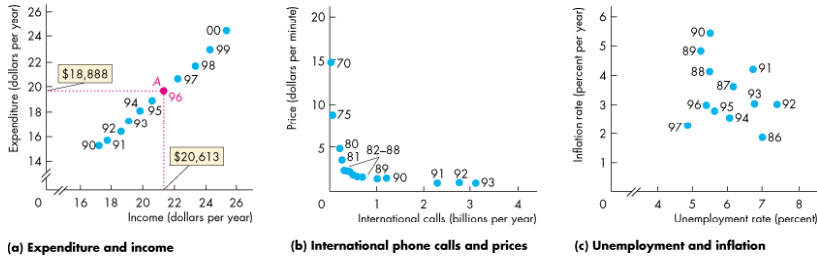
## Graphing Data

### Scatter Diagrams

- A **scatter diagram** plots the value of one variable on the x-axis and the value of another variable on the y-axis.
- A scatter diagram can make clear the relationship between two variables.
- The three scatter diagrams on the next slide show examples of variables that move in the same direction, in opposite directions, and in no particular relationship to each other.

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# Graphing Data



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# Graphs Used in Economic Models

- Graphs are used in economic models to show the relationship between variables.
- The patterns to look for in graphs are the four cases in which:
  - Variables move in the same direction
  - Variables move in opposite directions
  - Variables have a maximum or a minimum (extremals)
  - Variables are unrelated

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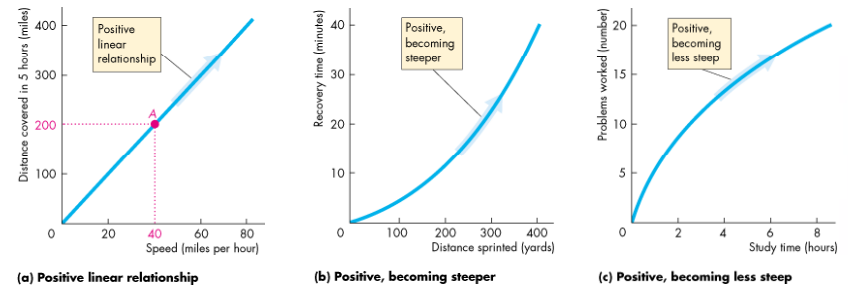
# Graphs Used in Economic Models

## Variables That Move in the Same Direction

- A relationship between two variables that move in the same direction is called a **positive relationship** or a **direct relationship**.
- A line that slopes upward shows a positive relationship.
- A relationship shown by a straight line is called a **linear relationship**.
- The three graphs on the next slide show positive relationships.

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# Graphs Used in Economic Models



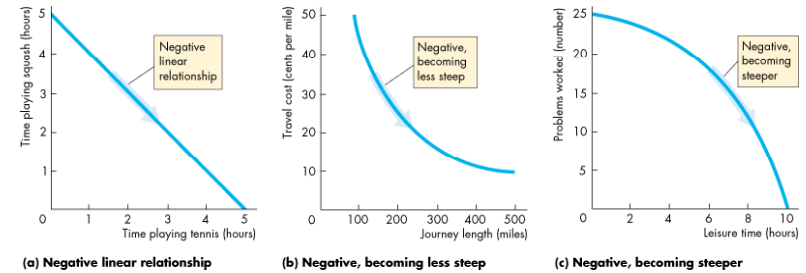
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## Graphs Used in Economic Models

### Variables That Move in Opposite Directions

- A relationship between two variables that move in opposite directions is called a **negative relationship** or an **inverse relationship**.
- A line that slopes downward shows a negative relationship.
- The three graphs on the next slide show negative relationships.

## Graphs Used in Economic Models

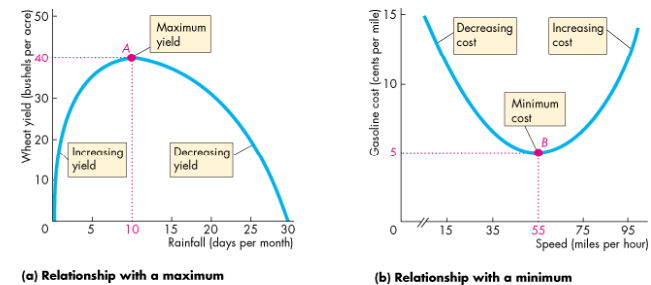


## Graphs Used in Economic Models

### Extremals: Variables That Have a Maximum or a Minimum

- The two graphs on the next slide show relationships that have a maximum and a minimum.
- These relationships are positive over part of their range and negative over the other part.

## Graphs Used in Economic Models



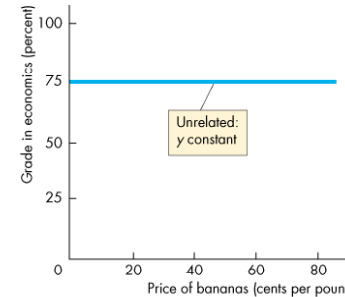
## Graphs Used in Economic Models

### Variables That are Unrelated

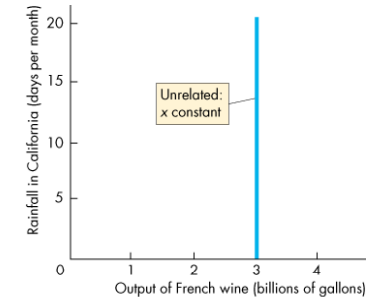
- Sometimes we want to emphasize that two variables are unrelated.
- The two graphs on the next slide show examples of variables that are unrelated.

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## Graphs Used in Economic Models



(a) Unrelated:  $y$  constant

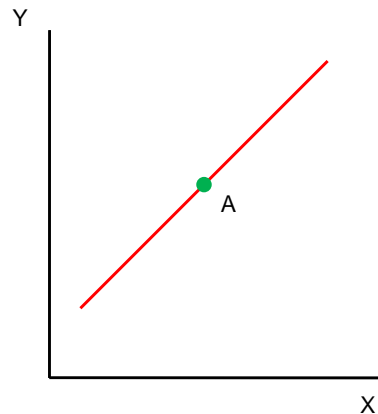


(b) Unrelated:  $x$  constant

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## Movements versus Shifts of Curves

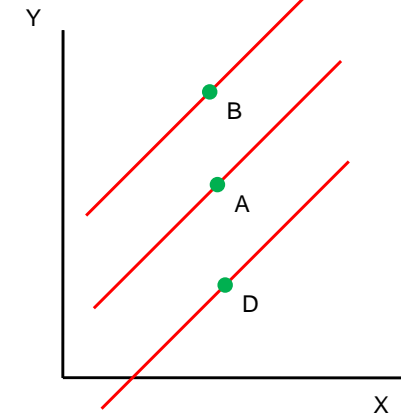
- What causes a movement along a graph, what causes a curve to shift?
- The easiest way to think about it is for a straight line (linear) graph
- Changes in  $Y$  or  $X$  causes a point like  $A$  to move along the curve



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## Movements versus Shifts of Curves

- Anything else that affects the relationship between  $Y$  and  $X$ , e.g. " $c$ " will shift the curve
- Suppose that there is a positive relationship between " $c$ " and  $Y$ , then an increase in  $c$  will shift the curve up from  $A$  to a point like  $B$
- If a negative relationship exists between " $c$ " and  $Y$ , an increase in  $c$  shifts the curve downwards from  $A$  to a point like  $D$



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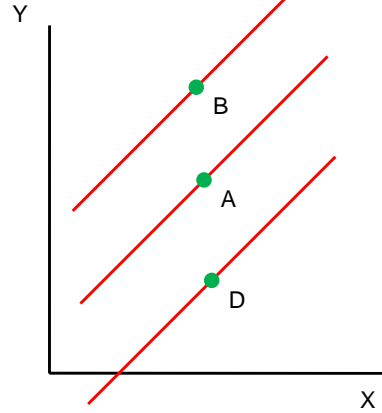
## Movements versus Shifts of Curves

- Algebraically, for a straight line:

$$Y = mX + c$$

where

- Y is the **dependent variable** (the thing we are looking to explain)
- X is the **independent variable** (the thing we are using to explain/ relating to Y)
- m is the **slope**
- c is the **intercept**
- Thus anything else that impacts Y (apart from X) will change the intercept, c, shifting the curve!



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## The Slope of a Relationship

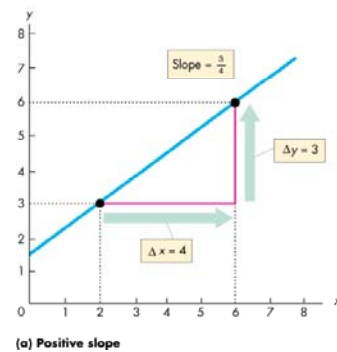
- The **slope** of a relationship is the change in the value of the variable measured on the y-axis divided by the change in the value of the variable measured on the x-axis.
- We use the Greek letter  $\Delta$  (capital delta) to represent “change in.”
- So  $\Delta y$  means the change in the value of the variable measured on the y-axis and  $\Delta x$  means the change in the value of the variable measured on the x-axis.
- The slope of the relationship is  $\Delta y/\Delta x$ .

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## The Slope of a Relationship

### The Slope of a Straight Line

- The slope of a straight line is constant.
- Graphically, the slope is calculated as the “rise” over the “run.”
- The slope is positive if the line is upward sloping.

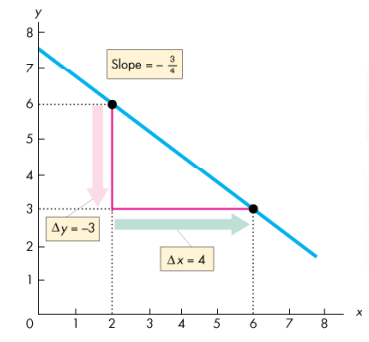


(a) Positive slope

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## The Slope of a Relationship

- The slope is negative if the line is downward sloping.



(b) Negative slope

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## The Slope of a Relationship

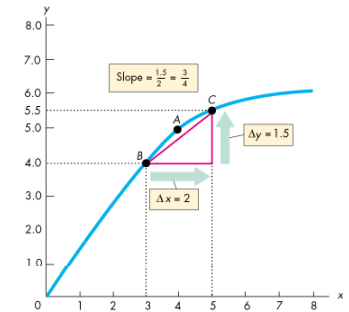
- The Slope of a Curved Line

- The slope of a curved line at a point varies depending on where along the curve it is calculated.
- We can calculate the slope of a curved line either at a point or across an arc.

## The Slope of a Relationship

### Slope Across an Arc

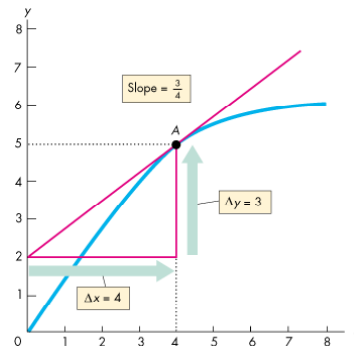
- The *average* slope of a curved line across an arc is equal to the slope of a straight line that joins the endpoints of the arc.
- Here, we calculate the average slope of the curve along the arc *BC*.



## The Slope of a Relationship

### Slope at a Point

- The slope of a curved line at a point is equal to the slope of a straight line that is the tangent to that point.
- Here, we calculate the slope of the curve at point *A*.



## More on Calculating Slopes

### Slope of a Linear Function

- Consider a linear function:  
 $y = f(x) = mx + c$
- Suppose that the pairs  $(x_0, y_0)$  and  $(x_1, y_1)$  both represent arbitrary points on the line. Then the ratio:  
$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$
is called the slope.
- $c$  is called the intercept
- Question: What is the slope of a line that goes through the points  $(4, 6)$  and  $(0, 7)$ ?

## Solution...

- Take the points (4,6) and (0,7)
- If we plug into the formula on the previous page:

$$\frac{\Delta y}{\Delta x} = m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$= \frac{7 - 6}{0 - 4} = \frac{1}{(-4)} = -\frac{1}{4}$$

- Hence the slope of a line that goes through the points (4,6) and (0,7) is -1/4.

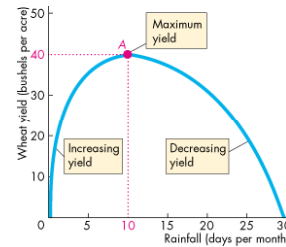
## Another Practice Problem... (Solution next page)

### Centigrade and Fahrenheit

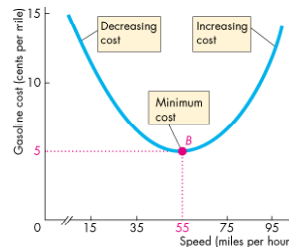
- Let C denote the temperature in degrees Centigrade and let F denote the temperature in degrees Fahrenheit.
- We know that 0° Centigrade is 32° Fahrenheit is the freezing temperature of water, and that 100° Centigrade or 212° Fahrenheit is the boiling point for water.
- What is the equation that relates degrees Fahrenheit to degrees Centigrade? What is the inverse function, i.e. the equation that relates degrees Centigrade to degrees Fahrenheit?

## How to find Extremals ...?

- Take our two graphs from earlier...
- ... how would we find any minima or maxima that exist?
- Answer:
  - Look for “stationary points”, i.e. where the slopes of the curves are zero!



(a) Relationship with a maximum



(b) Relationship with a minimum

## How to find Extremals ...?

- To find any maxima or minima, use the following methodology:
  1. Calculate the slope of the function  $y=f(x)$ , i.e.  $\Delta y/\Delta x$
  2. Set  $\Delta y/\Delta x=0$  and find values of x where the slope equals zero!
  3. Check around those points found in step 2.
    1. If the slope is increasing then decreasing, it is a maximum
    2. If the slope is decreasing then increasing, then it is a minimum.

## Slope Terminology

- For a linear function:  $y = f(x) = mx + c$ , the slope is easy to calculate:

$$\text{slope} = \frac{\Delta y}{\Delta x} = m$$

- When the change,  $\Delta$ , is very (very, very, very, ...) small, i.e. as  $\Delta \rightarrow 0$ , we write the change in  $y$  relative to  $x$  as:

$$\frac{dy}{dx}$$

and call it the “**derivative**” of  $y$  with respect to  $x$

## Some Rules to Calculate Slopes

However, for **nonlinear functions**, use the following rules:

- The slope of a constant = 0. e.g. slope of  $y=5$  is 0.
- The slope of a variable to the  $k^{\text{th}}$  power, e.g.  $y=x^k$  is:

$$\frac{\Delta y}{\Delta x} = \frac{dy}{dx} = kx^{k-1},$$

For example: the slope of  $y=x^4$  is:  $\frac{dy}{dx} = 4x^3$

- $y=Ax^k$  is  $\frac{dy}{dx} = Akx^{k-1}$ , for example: for  $y=5x^4$ , the slope

$$\text{is: } \frac{\Delta y}{\Delta x} = 5 * 4x^3 = 20x^3$$

## Some Rules to Calculate Slopes (cont.)

The slope of:

- Combining (3) and (1), we get:

- For  $y = Ax^k + c$ ,  $\frac{\Delta y}{\Delta x} = Akx^{k-1}$ ,

e.g. for  $y=5x^4 + 10$ ,

$$\frac{\Delta y}{\Delta x} = 5 * 4x^3 = 20x^3$$

- For  $y = Ax^k + Bx^n$ ,  $\frac{\Delta y}{\Delta x} = Akx^{k-1} + Bnx^{n-1}$ ,

e.g. for  $y=5x^4 + 3x^5$

$$\frac{\Delta y}{\Delta x} = 5 * 4x^3 + 3 * 5x^4 = 20x^3 + 15x^4$$

## Some Rules to Calculate Slopes (cont.)

The slope of:

- If  $y$  is a function of several variables, e.g.  $y=f(x,s,t)$ , treat the other variables as **constant** when calculating the slope of  $y$  wrt  $x$ !

e.g.  $y=100 + 5x^4 + 10s^2 + 15t^3$

$$\frac{dy}{dx} = 5 * 4x^3 = 20x^3;$$

Similarly  $\frac{dy}{ds} = 10 * 2s = 20s;$

$$\frac{dy}{dt} = 15 * 3t = 45t$$

## Summary of Rules

Quick Summary :

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$x^m$	$mx^{m-1}$
$Ae^{bx}$	$Abe^{bx}$
$\ln x$	$(1/x)$
$a^x$	$a^x \ln a$

## The End

Things to take away:

- Movement versus shifts of curves
- How to calculate slopes
  - Derivative and how to calculate it