

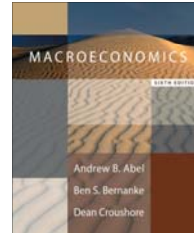
Business Cycles

Econ 402

Professor Yamin Ahmad

Lecture 2:

- Understanding and Interpreting Graphs in Economics



Some Tools of Analysis

- Graphs:
 - Movements
 - Shifts of Curves
- Extremals: Maxima and Minima

Note: These lecture notes are incomplete without having attended lectures

2

Graphing Data

- Economists use three types of graphs to reveal relationships between variables. They are:
 - Time-series graphs (- mostly in macroeconomics)
 - Cross-section graphs (- mostly in microeconomics)
 - Scatter diagrams (- in both, and in particular, econometrics)

Note: These lecture notes are incomplete without having attended lectures

3

Graphing Data

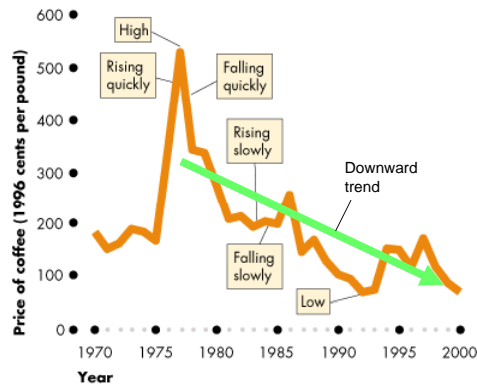
Time-Series Graphs

- A **time-series graph** measures time (for example, months or years) along the x-axis and the variable or variables in which we are interested along the y-axis.
- The time-series graph on the next slide shows the price of coffee between 1970 and 2000.
- The graph shows the level of the price, how it has changed over time, when change was rapid or slow, and whether there was any trend.

Note: These lecture notes are incomplete without having attended lectures

4

Graphing Data



Note: These lecture notes are incomplete without having attended lectures

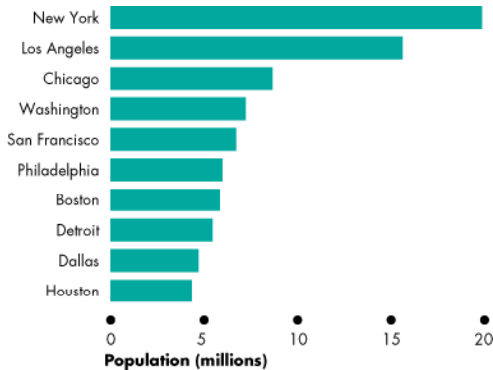
Graphing Data

Cross-Section Graphs

- A **cross-section graph** shows the values of a variable for different groups in a population at a point in time.
- The cross-section graph on the next slide enables you to compare the number of people who live in 10 metropolitan areas in the United States.

Note: These lecture notes are incomplete without having attended lectures

Graphing Data



Note: These lecture notes are incomplete without having attended lectures

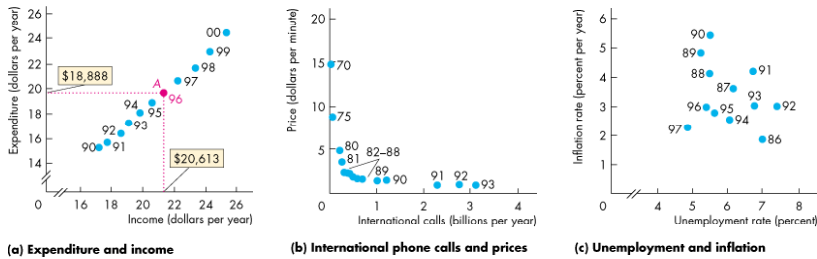
Graphing Data

Scatter Diagrams

- A **scatter diagram** plots the value of one variable on the x-axis and the value of another variable on the y-axis.
- A scatter diagram can make clear the relationship between two variables.
- The three scatter diagrams on the next slide show examples of variables that move in the same direction, in opposite directions, and in no particular relationship to each other.

Note: These lecture notes are incomplete without having attended lectures

Graphing Data



Note: These lecture notes are incomplete without having attended lectures

Graphs Used in Economic Models

- Graphs are used in economic models to show the relationship between variables.
- The patterns to look for in graphs are the four cases in which:
 - Variables move in the same direction
 - Variables move in opposite directions
 - Variables have a maximum or a minimum (extremals)
 - Variables are unrelated

Note: These lecture notes are incomplete without having attended lectures

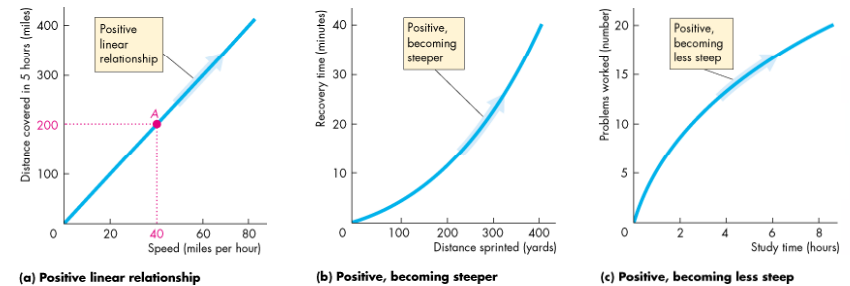
Graphs Used in Economic Models

Variables That Move in the Same Direction

- A relationship between two variables that move in the same direction is called a **positive relationship** or a **direct relationship**.
- A line that slopes upward shows a positive relationship.
- A relationship shown by a straight line is called a **linear relationship**.
- The three graphs on the next slide show positive relationships.

Note: These lecture notes are incomplete without having attended lectures

Graphs Used in Economic Models



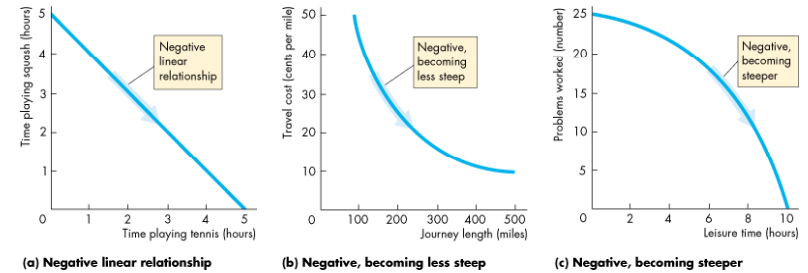
Note: These lecture notes are incomplete without having attended lectures

Graphs Used in Economic Models

Variables That Move in Opposite Directions

- A relationship between two variables that move in opposite directions is called a **negative relationship** or an **inverse relationship**.
- A line that slopes downward shows a negative relationship.
- The three graphs on the next slide show negative relationships.

Graphs Used in Economic Models

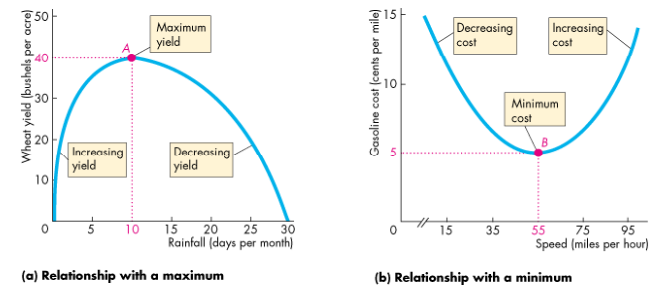


Graphs Used in Economic Models

Extremals: Variables That Have a Maximum or a Minimum

- The two graphs on the next slide show relationships that have a maximum and a minimum.
- These relationships are positive over part of their range and negative over the other part.

Graphs Used in Economic Models



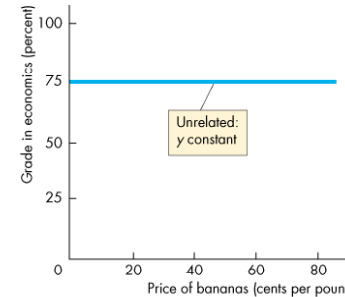
Graphs Used in Economic Models

Variables That are Unrelated

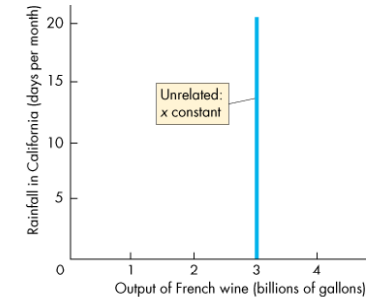
- Sometimes we want to emphasize that two variables are unrelated.
- The two graphs on the next slide show examples of variables that are unrelated.

Note: These lecture notes are incomplete without having attended lectures

Graphs Used in Economic Models



(a) Unrelated: y constant

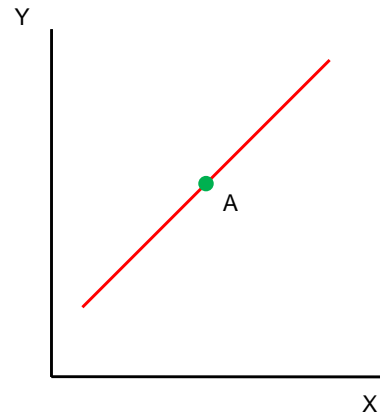


(b) Unrelated: x constant

Note: These lecture notes are incomplete without having attended lectures

Movements versus Shifts of Curves

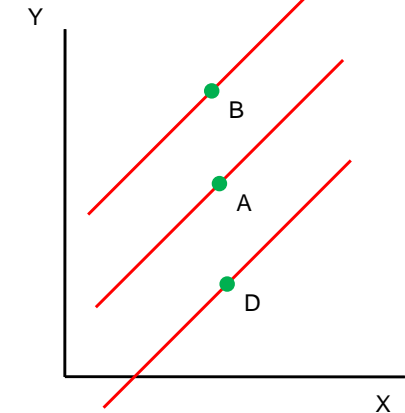
- What causes a movement along a graph, what causes a curve to shift?
- The easiest way to think about it is for a straight line (linear) graph
- Changes in Y or X causes a point like A to move along the curve



Note: These lecture notes are incomplete without having attended lectures

Movements versus Shifts of Curves

- Anything else that affects the relationship between Y and X , e.g. " c " will shift the curve
- Suppose that there is a positive relationship between " c " and Y , then an increase in c will shift the curve up from A to a point like B
- If a negative relationship exists between " c " and Y , an increase in c shifts the curve downwards from A to a point like D



Note: These lecture notes are incomplete without having attended lectures

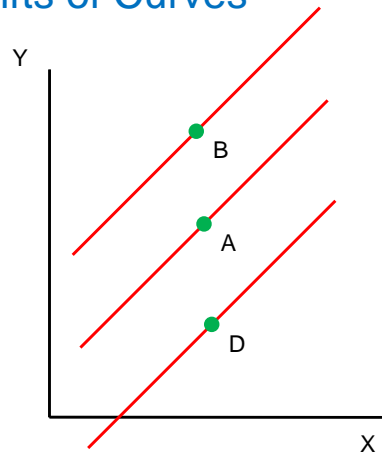
Movements versus Shifts of Curves

- Algebraically, for a straight line:

$$Y = mX + c$$

where

- Y is the **dependent variable** (the thing we are looking to explain)
- X is the **independent variable** (the thing we are using to explain/ relating to Y)
- m is the **slope**
- c is the **intercept**
- Thus anything else that impacts Y (apart from X) will change the intercept, c, shifting the curve!



Note: These lecture notes are incomplete without having attended lectures

The Slope of a Relationship

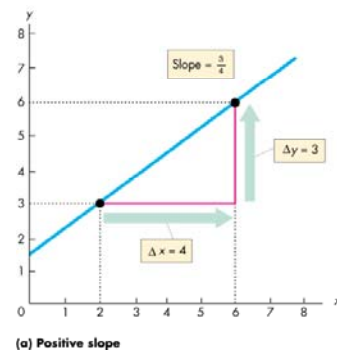
- The **slope** of a relationship is the change in the value of the variable measured on the y-axis divided by the change in the value of the variable measured on the x-axis.
- We use the Greek letter Δ (capital delta) to represent “change in.”
- So Δy means the change in the value of the variable measured on the y-axis and Δx means the change in the value of the variable measured on the x-axis.
- The slope of the relationship is $\Delta y/\Delta x$.

Note: These lecture notes are incomplete without having attended lectures

The Slope of a Relationship

The Slope of a Straight Line

- The slope of a straight line is constant.
- Graphically, the slope is calculated as the “rise” over the “run.”
- The slope is positive if the line is upward sloping.

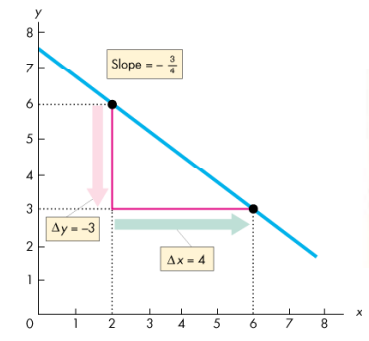


(a) Positive slope

Note: These lecture notes are incomplete without having attended lectures

The Slope of a Relationship

- The slope is negative if the line is downward sloping.



(b) Negative slope

Note: These lecture notes are incomplete without having attended lectures

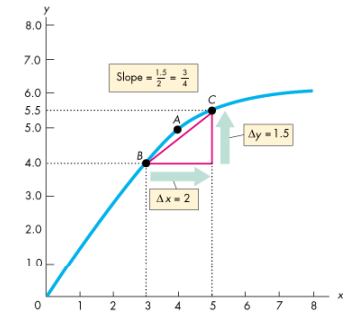
The Slope of a Relationship

- The Slope of a Curved Line
 - The slope of a curved line at a point varies depending on where along the curve it is calculated.
 - We can calculate the slope of a curved line either at a point or across an arc.

The Slope of a Relationship

Slope Across an Arc

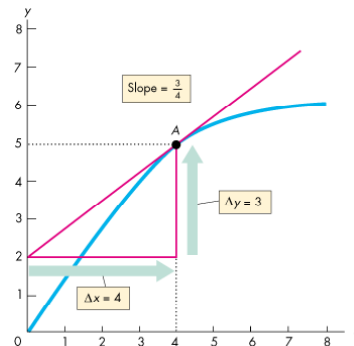
- The *average* slope of a curved line across an arc is equal to the slope of a straight line that joins the endpoints of the arc.
- Here, we calculate the average slope of the curve along the arc BC.



The Slope of a Relationship

Slope at a Point

- The slope of a curved line at a point is equal to the slope of a straight line that is the tangent to that point.
- Here, we calculate the slope of the curve at point A.



More on Calculating Slopes

Slope of a Linear Function

- Consider a linear function:

$$y = f(x) = mx + c$$
- Suppose that the pairs (x_0, y_0) and (x_1, y_1) both represent arbitrary points on the line. Then the ratio:

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$
 is called the slope.
- c is called the intercept
- Question: What is the slope of a line that goes through the points $(4, 6)$ and $(0, 7)$?

Solution...

- Take the points (4,6) and (0,7)
- If we plug into the formula on the previous page:

$$\frac{\Delta y}{\Delta x} = m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$= \frac{7 - 6}{0 - 4} = \frac{1}{(-4)} = -\frac{1}{4}$$

- Hence the slope of a line that goes through the points (4,6) and (0,7) is -1/4.

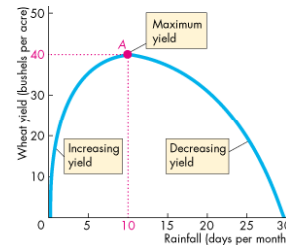
Another Practice Problem... (Solution next page)

Centigrade and Fahrenheit

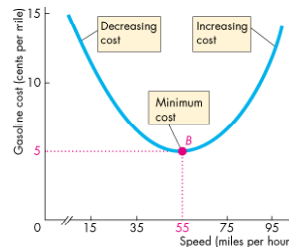
- Let C denote the temperature in degrees Centigrade and let F denote the temperature in degrees Fahrenheit.
- We know that 0° Centigrade is 32° Fahrenheit is the freezing temperature of water, and that 100° Centigrade or 212° Fahrenheit is the boiling point for water.
- What is the equation that relates degrees Fahrenheit to degrees Centigrade? What is the inverse function, i.e. the equation that relates degrees Centigrade to degrees Fahrenheit?

How to find Extremals ...?

- Take our two graphs from earlier...
- ... how would we find any minima or maxima that exist?
- Answer:
 - Look for “stationary points”, i.e. where the slopes of the curves are zero!



(a) Relationship with a maximum



(b) Relationship with a minimum

How to find Extremals ...?

- To find any maxima or minima, use the following methodology:
 1. Calculate the slope of the function $y=f(x)$, i.e. $\Delta y/\Delta x$
 2. Set $\Delta y/\Delta x=0$ and find values of x where the slope equals zero!
 3. Check around those points found in step 2.
 1. If the slope is increasing then decreasing, it is a maximum
 2. If the slope is decreasing then increasing, then it is a minimum.

Slope Terminology

- For a linear function: $y = f(x) = mx + c$, the slope is easy to calculate:

$$\text{slope} = \frac{\Delta y}{\Delta x} = m$$

- When the change, Δ , is very (very, very, very, ...) small, i.e. as $\Delta \rightarrow 0$, we write the change in y relative to x as:

$$\frac{dy}{dx}$$

and call it the “**derivative**” of y with respect to x

Some Rules to Calculate Slopes

However, for **nonlinear functions**, use the following rules:

- The slope of a constant = 0. e.g. slope of $y=5$ is 0.
- The slope of a variable to the k^{th} power, e.g. $y=x^k$ is:

$$\frac{\Delta y}{\Delta x} = \frac{dy}{dx} = kx^{k-1},$$

For example: the slope of $y=x^4$ is: $\frac{dy}{dx} = 4x^3$

- $y=Ax^k$ is $\frac{dy}{dx} = Akx^{k-1}$, for example: for $y=5x^4$, the slope

$$\text{is: } \frac{\Delta y}{\Delta x} = 5 * 4x^3 = 20x^3$$

Some Rules to Calculate Slopes (cont.)

The slope of:

- Combining (3) and (1), we get:

- For $y = Ax^k + c$, $\frac{\Delta y}{\Delta x} = Akx^{k-1}$,

e.g. for $y=5x^4 + 10$,

$$\frac{\Delta y}{\Delta x} = 5 * 4x^3 = 20x^3$$

- For $y = Ax^k + Bx^n$, $\frac{\Delta y}{\Delta x} = Akx^{k-1} + Bnx^{n-1}$,

e.g. for $y=5x^4 + 3x^5$

$$\frac{\Delta y}{\Delta x} = 5 * 4x^3 + 3 * 5x^4 = 20x^3 + 15x^4$$

Some Rules to Calculate Slopes (cont.)

The slope of:

- If y is a function of several variables, e.g. $y=f(x,s,t)$, treat the other variables as **constant** when calculating the slope of y wrt x !

e.g. $y=100 + 5x^4 + 10s^2 + 15t^3$

$$\frac{dy}{dx} = 5 * 4x^3 = 20x^3;$$

Similarly $\frac{dy}{ds} = 10 * 2s = 20s;$

$$\frac{dy}{dt} = 15 * 3t = 45t$$

Summary of Rules

Quick Summary :

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x^m	mx^{m-1}
Ae^{bx}	Abe^{bx}
$\ln x$	$(1/x)$
a^x	$a^x \ln a$

The End

Things to take away:

- Movement versus shifts of curves
- How to calculate slopes
 - Derivative and how to calculate it