

Real Business Cycles

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Overview

Neoclassical Theory:

- Perfect Competition with Rational Expectations (RE);
- Imperfect Information
- Nominal shocks are sources of cycles

Real Business Cycle Theory

- Frictionless Perfectly Competitive Economies
- Complete Markets
- Real Shocks
- Cycles arise through reactions of optimizing agents to real disturbances

Overview

New Keynesian Theory:

- Rational Expectations (RE)
- Imperfect Competition
- Nominal and real shocks

Basic Features of RBC Models

- 1 Representative agent framework: allows us to circumvent aggregation problems
- 2 Firms and households optimize an explicit objective function, subject to resource and technology constraints
- 3 The cycle is driven by exogenous shocks to technology that shift the production function around
 - The impact of these shocks on output are amplified via an intertemporal labor substitution mechanism.
- 4 All agents have rational expectations
- 5 Markets are complete and continuously clear.

Basic Features of RBC Models

Actual 'business cycles' are generated as shocks propagate through the economy. This can take several forms:

- 1 Agents seek to smooth consumption over time
 - Hence an increase in output also leads to an increase in investment and capital stock
- 2 Lags in the investment process can result in a shock today affecting investment in the future, and hence future output
- 3 Individuals will tend to substitute leisure intertemporally in response to changes in wages - work harder when wages are temporarily higher, and compensate when wages fall to previous levels.
- 4 Firms may use inventories to meet unexpected changes in demand.

Households

- Economy populated with identical, infinitely lived agents that produce a single good
- No frictions or transactions costs
- For simplicity, abstract from money and government

Each agent maximizes:

$$U_t = \max E_t \left[\sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \right], \quad 0 < \beta < 1 \quad (1)$$

where β , c_t and l_t are a discount factor, consumption and leisure and E is the expectations operator.

Technology

- Constant-returns-to-scale (CRS) production function
- Law of motion of the capital stock over time

Production Function is:

$$y_t = z_t f(k_t, n_t) \quad (2)$$

where y_t is output, k_t is capital carried over from previous period and n_t is labor.

z_t represents a productivity (TFP) shock and is the source of uncertainty in the model.

- There is a probability distribution governing z_t , and observations of z_t are simply draws from that probability distribution through time - hence the stochastic nature of z_t .

Law of Motion for Capital Stock

The capital stock evolves according to:

$$k_{t+1} = (1 - \delta) k_t + i_t \quad (3)$$

where δ is the rate of depreciation, i_t is gross investment.

- In a one good model, part of output that is not consumed becomes part of the capital stock the next period.

Resource Constraints

The resource constraints faced by an agent every period are:

- Resource constraint:

$$c_t + i_t = y_t$$

- Time constraint:

$$n_t + l_t = 1 \tag{4}$$

Aggregation

- Because all agents are identical, aggregation is fairly simple: we simply solve for the equilibrium quantities and prices by solving the representative agent's optimization problem.

Optimization Problem for Households

- Since expectations are rational, households know the probability distribution generating z_t and so the household's maximization problem involves maximizing equation (1) subject to equations (2) - (4).

Households:- Lagrangean in period t is given by:

$$\mathcal{L} = u(c_t, (1 - n_t)) + \lambda_t [z_t f(k_t, n_t) - c_t - k_{t+1} + (1 - \delta) k_t]$$

First order Conditions

The first order conditions are:

$$u_1(c_t, l_t) - \lambda = 0 \quad (5)$$

$$u_2(c_t, l_t) + \lambda_t z_t f_2(k_t, n_t) = 0 \quad (6)$$

$$-\lambda_t + E_t \beta \lambda_{t+1} [z_t f_1(k_t, n_t) - (1 - \delta)] = 0 \quad (7)$$

$$z_t f(k_t, n_t) + (1 - \delta) k_t = c_t + k_{t+1} \quad (8)$$

Example:

Suppose that:

- The period utility function is log-linear:

$$u(\cdot) = \theta \ln c_t + (1 - \theta) \ln(1 - n_t) \quad (9)$$

- and that the production function is Cobb-Douglas:

$$z_t f(\cdot) = z_t k_t^\alpha n_t^{1-\alpha} \quad (10)$$

- The rate of depreciation is 1, i.e. $\delta = 1$.

First Order Conditions

$$\mathcal{L} = \sum_{j=0}^{\infty} \beta^j \{ (\theta \ln c_{t+j} + (1-\theta) \ln(1-n_{t+j})) + \dots \\ \dots + \lambda_{t+j} [z_{t+j} k_{t+j}^{\alpha} n_{t+j}^{1-\alpha} - c_{t+j} - k_{t+j+1} + (1-\delta) k_{t+j}] \}$$

First order conditions:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \frac{\theta}{c_t} - \lambda_t = 0 \quad (11)$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = -\frac{1-\theta}{1-n_t} + \lambda_t (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha} = 0 \quad (12)$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\lambda_t + \beta E_t [\lambda_{t+1} \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1-\delta)] = 0 \quad (13)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} \Rightarrow c_t + k_{t+1} = z_t k_t^{\alpha} n_t^{1-\alpha} + (1-\delta) k_t \quad (14)$$

The Labor-Leisure Tradeoff

We can use equations (11) and (12) to derive the labor-leisure tradeoff as follows:

$$\begin{aligned}\lambda_t &= \frac{\theta}{c_t} \\ \implies \frac{1-\theta}{1-n_t} &= \frac{\theta}{c_t} (1-\alpha) z_t k_t^\alpha n_t^{-\alpha} \\ \implies \frac{c_t}{l_t} &= \frac{\theta}{1-\theta} (1-\alpha) z_t k_t^\alpha n_t^{-\alpha}\end{aligned}$$

- In other words, the opportunity cost of an additional hour of labor is the amount of foregone consumption, which amount to the marginal product of labor balanced by the relative preference for consumption versus leisure.

The Consumption Euler Equation

We can use equations (11) and (13) to derive the consumption-Euler equation as follows:

$$\begin{aligned}\lambda_t &= \beta E_t \left[\alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1 - \delta) \right] \\ \implies \frac{\theta}{c_t} &= \beta E_t \left[\frac{\theta}{c_{t+1}} \left(\alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1 - \delta) \right) \right]\end{aligned}$$

- Under some distributional assumptions about consumption and productivity, we can separate the expression above into:

$$E_t \left(\frac{c_{t+1}}{c_t} \right) = \beta E_t \left(\alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1 - \delta) \right)$$

- Since $MPK - \delta = r$

$$E_t \left(\frac{c_{t+1}}{c_t} \right) = \beta (1 + r_t)$$

Solving the Model

Given the earlier assumptions we made regarding the explicit form for the utility and production functions, we can derive analytical expressions for the optimal choices.

- The income and substitution effect of a wage change cancel so that labor/leisure is constant at the solution.
- For the remaining variables, we can use the method of undetermined coefficients to solve for the capital stock and consumption, i.e.:

$$c_t^* = [1 - \alpha\beta] z_t k_t^\alpha n_t^{1-\alpha} \quad (15)$$

$$k_{t+1}^* = \alpha\beta z_t k_t^\alpha n_t^{1-\alpha} \quad (16)$$

- These time paths satisfy the first order conditions and thus represent optimal decision rules for agents in the economy.

Linearization

- For general utility and production functions, we may not be able to find an analytical solution, i.e. one that we can write down with paper and pencil methods.
- In this case, we typically linearize the first order conditions using a Taylor approximation around the steady state.
- Having linearized the system, we can then solve for the time path of the endogenous variables, c_t, k_{t+1}, y_t, i_t etc.

Calibration

- The next step is calibrating the model.
- This involves choosing specific values for the parameters. In the example earlier, this means picking out values for β , δ , α and θ .
- Choice of values for these parameters is typically done with reference to previous econometric studies, and matching to data econometrically, based on the type of data, the frequency, etc.

Simulations

- Artificial data is then generated from the model.
- This involves specifying a stochastic process for the technology parameter, e.g.

$$z_t = \rho z_{t-1} + \varepsilon_t$$

where ε_t represents a shock to productivity. ε_t could be from a Normal distribution, or from some other parametric distribution.

- Having specified the process for z_t , we then draw a series of shocks and then feed these shocks into the model to generate time paths for c_t, k_t etc.
- We then compare the model by examining the moments of the artificial model generated data, with actual data.

Impulse Response Functions

- Many studies examine the *impulse response function (IRF)* from the calibrated model
- This allows us to examine how different endogenous variables would respond to shocks hitting the economy
- Moreover, it allows us to see how the shocks propagate and allows us to measure
 - How long it takes to have an impact
 - How long the shock essentially lasts for (in terms of its impact on these variables)
 - Whether the system "recovers".