The Basic Structure of New Keynesian Models

Part II - Deriving the New Keynesian Phillips Curve

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Final Goods Firm

- The final goods firm produces a final consumption good, Y_t, at time t, using the intermediate goods produced by other firms as an input.
- It combines the continuum of intermediate goods, $j \in [0,1]$ using a constant returns to scale production technology:
- $$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \quad (1)$$
- where $Y_{j,t}$ is the quantity of intermediate good j used as the input.
- θ is the elasticity of substitution between the different intermediate goods, and it is constant.
- The final good firm takes the output price (of the final consumption good) and the input price (of the intermediate goods) as given and maximizes profits.

Final Goods Firm's problem:

The way to tackle the problem is to derive the aggregate demand curve (for each of the inputs) and price index before doing the full optimization problem. This is done by decomposing the problem into an:

1. Optimal *intratemporal* problem, which consists of the dual problem: minimize costs for a fixed expenditure, E.
2. Optimal *intertemporal* problem, which consists of the primal problem: optimal intertemporal allocation of the expenditure, E.

Final Goods Firm's problem:

- The final goods firm maximizes profits (or equivalently, minimizes costs), taking the final goods price, P_t and the prices of the intermediate goods, P_{jt} as given.
- The Lagrangean representing the cost minimization problem for the final goods firm (dropping t subscripts) to produce an amount of output $\bar{Y}$ is:

$$\mathcal{L} = \int_0^1 P_j Y_j dj - \lambda \left[\left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} - \bar{Y} \right]$$

Final Goods Firm's problem:

The first order condition is:

$$P_j - \lambda Y_j^{\left(\frac{\theta-1}{\theta}-1\right)} \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}-1} = 0$$

$$\implies P_j = \lambda Y_j^{-\frac{1}{\theta}} \left(\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj \right)^{\frac{1}{\theta-1}}$$

From the constraint, since $\int_0^1 Y_j^{\frac{\theta-1}{\theta}} dj = \bar{Y}^{\frac{\theta-1}{\theta}}$. Hence:

$$\begin{aligned} P_j &= \lambda Y_j^{-\frac{1}{\theta}} \\ &\implies P_j Y_j^{\frac{1}{\theta}} = \lambda \end{aligned}$$

or equivalently

$$P_j Y_j = \lambda Y_j^{\frac{\theta-1}{\theta}}$$

Final Goods Firm's problem:

From here, it is possible to show that:

$$P \equiv \lambda = \left(\int_0^1 P_j^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \tag{2}$$

- Hence λ represents the marginal cost of the bundle and it equals the price level.

Final Good Firm's problem:

- Solving the profit maximization problem for the firm yields (... after some algebra ...) the set of demand schedules for the individual intermediate goods:

$$Y_{jt} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} Y_t \tag{3}$$

- Equation (3) above represents the demand for individual (intermediate) goods from intermediate goods firm j by the final goods firm.
- We also obtain the zero profit condition which we saw above in equation (2):

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dz \right)^{\frac{1}{1-\theta}} \tag{4}$$

Intermediate Goods Firm's Problem

- Intermediate good $j \in (0,1)$, is produced by a monopolist who utilizes a Cobb-Douglas production technology:

$$Y_{j,t} = Z_t K_t^{1-\gamma} N_{j,t}^{\gamma} \tag{5}$$

  where Z_t is a productivity shock, K_t and $N_{j,t}$ are the amounts of capital and composite labor services employed by firm j.
- These intermediate firms act competitively with regards to input prices, and take wages take wages and the rental cost of capital as given when choosing the optimal amounts of labor.

Optimization Problem for Intermediate Goods Firms

- Hence the optimization problem for a representative intermediate goods firm can be written as (dropping the j subscripts):

$$TC \equiv \min_{K_t N_t} R_t^k K_t + W_t N_t \text{ subject to } Z_t K_t^{\alpha} N_t^{1-\alpha} \geq \overline{Y}_t$$

- The first order conditions yield the following labor demand equation:

$$R_t^k = \alpha \mu Z_t K_t^{\alpha-1} N_t^{1-\alpha} \quad (6)$$

$$W_t = (1-\alpha)\phi Z_t K_t^{\alpha} N_t^{-\alpha} \quad (7)$$

$$\overline{Y}_t = Z_t K_t^{\alpha} N_t^{1-\alpha} \quad (8)$$

Optimal K and L

- Solving for the optimal values of K_t^* and N_t^*, we obtain:

$$K_t^* = \left[\left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t^k}\right]^{1-\alpha}\frac{\bar{Y}_t}{Z_t}$$

$$N_t^* = \left[\left(\frac{1-\alpha}{\alpha}\right)\frac{R_t^k}{W_t}\right]^{\alpha}\frac{\bar{Y}_t}{Z_t}$$

- Plugging K_t^* and N_t^* back into the objective function, yields the Total (nominal) Cost function $TC(R_t^k, W_t, \bar{Y}_t)$:

$$TC = \frac{\left(R_t^k\right)^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \cdot \frac{\bar{Y}_t}{Z_t} \tag{9}$$

Marginal Costs

- To get marginal costs, we can differentiate the total cost function in equation (9) with respect to $\bar{Y}$:
- i.e. nominal marginal costs equal:

$$P_t s_t = \frac{1}{\Phi} Z_t^{-1} \left(R_t^k \right)^{\alpha} W_t^{1-\alpha} \tag{10}$$

- and real marginal costs (dividing through by the price level) equals:

$$s_t = \frac{1}{\Phi} Z_t^{-1} \left(r_t^k \right)^{\alpha} \omega_t^{1-\alpha}$$

where $\Phi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$, $r_t^k = \frac{R_t^k}{P_t}$ and $\omega_t = \frac{W_t}{P_t}$

General Framework: Calvo (1983) contracts

1. Firms every period get to set a new price with probability $1-\gamma$.
2. However, when they set their price, they do not know when they will be able to reoptimize their price level next. As such, the expected length of time that they expect the price to be in effect can be calculated as:

$$
\begin{aligned}
E\left(\tau | P_{j,t+\tau}\right) &= \left(1-\gamma\right).1 + \left(1-\gamma\right)\gamma.2 + \dots \\
&\quad \dots + \left(1-\gamma\right)\gamma^{t-1}.t + \dots \\
&= \frac{1}{1-\gamma}
\end{aligned}
$$

3. Hence in any given period, the fraction of firms with prices set k periods ago is: $\omega_k = \left(1-\gamma\right)\gamma^k$.
4. Notice that there is some positive probability that a Calvo contract will last an arbitrarily long amount of time.

Strategy for Deriving the NK Phillips Curve

1. Derive the price that firms who can reoptimize prices set, i.e. $\tilde{P}_t$. Log-linearize around the steady state.
2. Derive the aggregate pricing equation, P_t as a function of the firms that can reset prices. Log-linearize around its steady state.
3. Replace marginal costs with output gap terms.

Price Setting

- Consider the problem faced by a firm who can reoptimize their price. They maximize:

$$\max_{\tilde{P}_t} E_t \sum_{k=0}^{\infty} (\gamma\beta)^k V_{t+k} \left[P_{j,t+k} Y_{j,t+k} - TC_{t+k} \left(Y_{j,t+k} \right) \right]$$

- subject to equation (3) and (10), i.e.:

$$\begin{aligned} Y_{jt} &= \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} Y_t \\ \text{and } P_t s_t &= \frac{1}{\Phi} Z_t^{-1} \left(R_t^k \right)^{\alpha} W_t^{1-\alpha} \end{aligned}$$

- TC is given by equation (9) and V in the equation above represents the marginal value of a dollar.

First order conditions

- When you solve the first order conditions, and after some hefty algebra manipulations, you get:

$$E_t \left\{ \sum_{k=0}^{\infty} (\gamma\beta)^k \left(\frac{V_{t+k}}{\tilde{P}_{j,t}} \right) (1-\theta)\, Y_{j,t+k} \left[\tilde{P}_{j,t} - \mu_p P_{t+k} s_{t+k} \right] \right\} = 0 \tag{11}$$

where $\mu_p = \frac{\theta}{\theta - 1} > 1$, is the monopoly markup.

- Notice that in equation (11) above, for the expression to be equal to zero, the term in the square bracket has to equal zero (since everything else is positive).
- As such equation (11) states that:

$$\begin{aligned} \tilde{P}_{j,t} &= \mu_p P_{t+k} s_{t+k} \\ &= \mu_p MC_{t+k} \end{aligned}$$

- which says that firms set prices as a markup over nominal marginal costs.

Log-linearizing the Intermediate Firm's Pricing Equation

- When we log-linearize equation (11), we obtain:

$$\frac{\tilde{P}_t - \bar{P}}{\bar{P}} = (1 - \gamma\beta)\, E_t \sum_{k=0}^{\infty} (\gamma\beta)^k \left(\frac{MC_{t+k} - M\bar{C}}{M\bar{C}} \right)$$

- Taking logs, this can be shown to be:

$$\tilde{p}_t - p_t = (1 - \gamma\beta)\, mc\,(.)_t + \gamma\beta \left[E_t \left(\tilde{p}_{t+1} \right) - p_t \right] \qquad (12)$$

Deriving the Aggregate Price Level

- Notice from equation (4) that the aggregate price can be written as:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta} dz \right)^{\frac{1}{1-\theta}} = \left[\sum_{k=0}^{\infty} \omega_k \left(\tilde{P}_{t-k}^{1-\theta} \right) \right]^{\frac{1}{1-\theta}} \qquad (13)$$

- Lagging the equation above once and inserting it back in, we can re-write the aggregate price level as:

$$P_t = \left[(1-\gamma) \tilde{P}_t^{1-\theta} + \gamma P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \qquad (14)$$

Log linearizing the Aggregate Pricing equation

- Just like with the intermediate firm's pricing equation, once we log-linearize the aggregate pricing equation, we obtain:

$$p_t = (1-\gamma)\,\tilde{p}_t + \gamma p_{t-1} \tag{15}$$

- Finally, the New Keynesian Phillips Curve is obtained by eliminating the $\tilde{p}_t$ in equation (12) using equation (15):

$$\pi_t = \left[\frac{(1-\gamma)(1-\gamma\beta)}{\gamma}\right] mc_t\,(.) + \beta E_t \pi_{t+1}$$

- Combining with an output gap term, we obtain:

$$\pi_t = \beta E_t \pi_{t+1} + \left[\frac{(1-\gamma)(1-\gamma\beta)}{\gamma}\right] A x_t$$