

Midterm

Instructions: Please answer both questions. You should show your working and calculations for each applicable problem. Correct answers without working will get you relatively few points. Make sure to put your name and id number on the top of every single piece of paper that you turn in. Also, please write in a page number at the bottom of every piece of paper that you turn in and assemble them sequentially before submitting.

Question 1: Optimization (50%)

Consider Robinson Crusoe stuck on his island. Robinson Crusoe lives for two periods (- essentially we're going to break time down into two periods) – today and tomorrow. Robinson has two choices that he can make. He can produce a single good (- he can fish), which he can consume. Alternatively, he can work on his suntan (leisure).

Robinson's key objective is to maximize his lifetime utility, which is an additively separable function of his utility every period, i.e.

$$\max U(c_t, l_t) = \sum_{j=0}^1 u(c_{t+j}, l_{t+j}) = u(c_t, l_t) + u(c_{t+1}, l_{t+1}) \quad (1)$$

where the utility function every period is given by: $u(c_t, l_t) = \theta \ln c_t + (1 - \theta) \ln l_t$. c_t and l_t represent consumption and leisure in period t respectively and you can assume that $\theta = \frac{1}{2}$. Suppose that

Robinson's production function is given by a constant returns to scale production function:

$$\begin{aligned} y_t &= f(k, l) \\ &= zk^{\frac{1}{2}}n_t^{\frac{1}{2}} \end{aligned} \quad (2)$$

where y_t is output (number of fish he gets), k is capital (- the fishing net he uses to get the fish) and n_t is labor. z is a productivity parameter (-essentially representing his productivity, which could depend on a number of different things, e.g. weather, tidal patterns etc). Finally, Robinson faces two resource constraints. Namely consumption has to equal output (i.e. he can only eat what he catches) and the sum of leisure and labor are normalized to equal one, i.e.

$$c_t = y_t \quad (3)$$

$$n_t + l_t = 1 \quad (4)$$

- a. [8 pts] Consider the period t . Write down the Lagrangean that represents Robinson Crusoe's optimization problem for this period.
- b. [6 pts] Derive the first order conditions (FOC) for the problem above. [Hint: Recall that the derivative of $\ln x$ equals: $\frac{d \ln x}{dx} = \frac{1}{x}$].
- c. [8 pts] By eliminating λ (or otherwise), write down an equation depicting the labor-leisure/consumption tradeoff, i.e. an equation that relates consumption to leisure, l . Providing economic intuition, interpret the equation that you have written. [Hint: If you can re-write the equation as saying $\frac{c_t}{l_t} = \dots$, you may find it easier to interpret].
- d. [10 pts] Solve for the optimal values of c_t, n_t, l_t and y_t . [Hint: you should find that they are functions of z, k and constants].
- e. [8 pts] Using your answers to part (d) above, show how consumption, labor and leisure would change if productivity, z , changes. What's the intuition for these results?
- f. [10 pts] Now consider the same problem for Robinson Crusoe, next period, $t+1$. Using your answers (a) to (e), or otherwise, derive the optimal solutions for consumption, labor and leisure in period $t+1$. How do your answers compare to what you calculated in part (d)? What is the intuition for the solutions here? [Hints and suggestion: 5 of the 10 points in this question are for the very last things I ask. Namely, how do the answers compare and the intuition behind it. Also, there is a VERY quick way to answer this question. My hint would be to look at equations (1)-(4).]

Question 2: IS-LM (50%)

Consider the following IS-LM model. First consider the demand side of the economy. The goods market is characterized by the following behavioral equations:

$$C = c_0 + c_1(Y - T) - c_2r$$

$$I = b_0 + b_1Y - b_2r$$

$$G = G_0$$

$$T = T_0$$

The money market is characterized by the demand and supply for money:

$$L(r, Y) = d_0 + d_1Y - d_2r$$

$$M = \bar{M}$$

Prices are simply denoted by P . Let the full employment level of output be denoted by \bar{Y} .

- a. [10 pts] For the set of equations characterizing the demand side of the economy above, write down which variables are the endogenous variables, which are the exogenous variables, and which variables are policy variables.
- b. [8 pts] For the same set of equations characterizing the demand side of the economy above, list the set of parameters. Given the structural relationships present between the different variables that are embedded in the equations above for the demand side, what can we say about the range of values that the individual parameters can take? That is, for each of the individual parameters that you list, also write down the range of values that it can take.
- c. [8 pts] Derive the IS relation using the equations above.
- d. [4 pts] Calculate the slope of the IS curve, i.e. $\frac{dr}{dY}$. What further restriction is required to ensure that a negative relationship exists between interest rates and output (i.e. to ensure that the IS curve slopes downwards)?
- e. [4 pts] Write down the equation for the LM curve. Rearrange your equation so that interest rates are a function of income. What is the slope of the LM curve?

- f. [6 pts] Consider the impact of a negative shock to the goods market. For example, the impact of a significant number of foreclosures in the housing market. Using diagrams, or otherwise, analyze the impact of this event within the IS-LM framework in the short run. Start by identifying which of the variables you indicated in part (a) might change. Then, outline the mechanism by which that change would impact both the goods and money market in the short run. What happens to output, real interest rates and prices/inflation in the short run?
- g. [3 pts] What would happen to investment in the scenario above in part (f)? [Hint: Check the equation for investment at the very beginning of this question in order to figure it out.]
- h. [4 pts] What kind of action should the government or the Federal Reserve take to correct and stabilize the economy? Pick **one** of the two. Then, using a diagram (preferred) or otherwise, outline the impact of such policy measures in the IS-LM model. What would happen to output, real interest rates and prices/inflation.
- i. [3 pts] Within the IS-LM framework, can we say anything about how long it takes for the shock to propagate through the economy, or how long it takes for the policy measures to actually have an impact? Discuss briefly.