



ECON 354 Money and Banking

Professor Yamin Ahmad

Lecture 3:

- Introduction to Interest Rates



Main Concepts

- Present Discounted Value (PDV)
- What is an interest rate?
 - Yield to maturity
 - A measure of an **Intertemporal Price**
 - Linked to the idea of Present Discounted Value!
 - Influences Savings/Lending Decisions of Agents in the Private Sector
- Interest Rates vs. Returns
- Nominal vs. Real interest rates and the Fisher Effect

Note: These lecture notes are incomplete without having attended lectures.



Present Discounted Value

- How does one compare payoffs (or streams of payoffs) from different points in time?
- Examples:
 - Lottery Win: \$1 Million today vs.. \$1000pm for next 10 years?
 - \$500 five years from now vs. \$800 ten years from now?
- Concept: Present Value (or Present Discounted Value – PDV short) is used to compare payoffs from different points in time
- Any time you compare payoffs at different points in time:
 - **USE PRESENT VALUE CALCULATIONS**

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Concept of Present Value

Simple Loan: Lender provides borrower with Principal, which borrower pays back to lender at a maturity date, along with additional interest payments

E.g. Simple loan of \$1 at 10% interest

Year	1	2	3	n
\$1	\$1.10	\$1.21	\$1.33	$\$1 \times (1 + i)^n$

PV of future \$1 = $\frac{\$1}{(1 + i)^n}$

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Four Types of Credit Instruments

1. **Simple loan:**
 - Principal + Interest paid to lender at given maturity date
2. **Fixed-payment loan:**
 - fixed payment (incorporating part of the principal and interest payment) paid over a period of time
3. **Coupon bond:**
 - Pays owner of bond a fixed (coupon) payment, until maturity when it pays off face (par) value
4. **Discount (zero coupon) bond:**
 - Bought at price below face value (discounted), and face value repaid at maturity.

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Yield to Maturity: Loans

Simple Loan:

Yield to maturity = interest rate that equates today's value with present value of all future payments

E.g. Car Loan (Simple Loan with $i = 10\%$)

$$\begin{aligned} \$100 &= \frac{\$110}{(1+i)} \\ \Rightarrow i &= \frac{\$110 - \$100}{\$100} = \frac{\$10}{\$100} = 0.1 = 10\% \end{aligned}$$

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Yield to Maturity: Loans (cont.)

Fixed Payment Loan

E.g. Fixed Payment Loan ($i = 12\%$)

$$\$1000 = \frac{\$126}{(1+i)} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + \dots + \frac{\$126}{(1+i)^{25}}$$

$$LV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \dots + \frac{FP}{(1+i)^n}$$

e.g. [Loan & Investment Formulas](#)

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Yield to Maturity: Bonds (cont.)

- **Coupon Bond:** Pays owner of bond a fixed (coupon) payment, until maturity when it pays off face (par) value

E.g. Coupon Bond (Coupon rate = 10% = C/F)

$$P = \frac{\$100}{(1+i)} + \frac{\$100}{(1+i)^2} + \frac{\$100}{(1+i)^3} + \dots + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}$$

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^{10}} + \frac{F}{(1+i)^{10}}$$

Consol: Fixed coupon payments of \$C forever

$$P = \frac{C}{i}; \quad i = \frac{C}{P}$$

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Yield to Maturity: Bonds (cont.)

- **Discount Bond:** Bought at price below face value (discounted), and face value repaid at maturity

E.g. **Discount Bond** ($P = \$900$, $F = \$1000$), one year

$$\begin{aligned} \$900 &= \frac{\$1000}{1+i} \\ \Rightarrow i &= \frac{\$1000 - \$900}{\$900} = 0.111 = 11.1\% \end{aligned}$$

Hence for a discount bond: $i = \frac{F - P}{P}$

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How are the price and yield to maturity related?

PRICE AND YIELD TO MATURITY

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Example

Price vs. Yields to Maturity	
Price of Bond (\$)	Yield to Maturity (%)
1200	7.13
1100	8.48
1000	10.00
900	11.75
800	13.81

Consider a bond with the following characteristics:
10% Coupon-Rate, Face Value = \$1000, Bond Maturing in 10 Years

Three Interesting Facts Emerge from the table above:

- When bond is at par, yield equals coupon rate
- Price and yield are negatively related
- Yield greater than coupon rate when bond price is below par value

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Current Yield

$$i_c = \frac{C}{P}$$

Two Characteristics

1. Is better approximation to yield to maturity, nearer price is to par and longer is maturity of bond
2. Change in current yield *always* signals change in same direction as yield to maturity

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Yield on a Discount Basis

$$i_{db} = \frac{F - P}{F} \times \frac{360}{(\text{number of days to maturity})}$$

Consider the following bill, $P = \$900$, $F = \$1000$ which has 1 year to maturity.

$$i_{db} = \frac{\$1000 - \$900}{\$1000} \times \frac{360}{365} = 0.099 = 9.9\%$$

Two Characteristics

1. Understates yield to maturity; longer the maturity, greater is understatement
2. Change in discount yield *always* signals change in same direction as yield to maturity

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Exercise: PDV, Yields

Try and calculate the answers to the following questions:

1. With an interest rate of 10%, a security pays \$1100 next year and \$1464 four years from now. Calculate the PDV of this security (to the nearest dollar).
2. A consol pays out \$20 annually. When interest rates are 5%, what is the price of the consol?
3. Suppose that the face value of a discount bond is \$1000. If the price of the discount bond today is \$926, what is the yield today (to the nearest percent)?
4. What is the current yield on a \$10000, 5% coupon bond selling for \$8000?

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Distinction Between Interest Rates and Returns

Rate of Return

Def: The **rate of return**, or simply, **return**, is the percentage change in the value of an asset.

$$R_{t+1} = \frac{C + P_{t+1} - P_t}{P_t} = i_c + g$$

where $i_c = \frac{C}{P_t}$ = current yield

and $g = \frac{P_{t+1} - P_t}{P_t}$ = capital gain/loss

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Relationship Between Interest Rates and Returns

One Year Returns on 10% Coupon Rate bonds of different Maturity when Interest Rate Rises from 10% to 20%

Years to Maturity when Bond is Purchased	Initial Current Yield (%)	Initial Price (\$)	Price Next Year (\$)	Rate of Capital Gain (%)	Rate of Return (Calc. as columns 2 + 5) (%)
30	10	1000	503	-49.7	-39.7
20	10	1000	516	-48.4	-38.4
10	10	1000	597	-40.3	-30.3
5	10	1000	741	-25.9	-15.9
2	10	1000	917	-8.3	1.7
1	10	1000	1000	0.0	10.0

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Maturity and the Volatility of Bond Returns

Key Findings from Table 2

1. Only bond whose return = yield is one with maturity = holding period
2. For bonds with maturity > holding period, $i \uparrow$ $P \downarrow$ implying capital loss
3. Longer is maturity, greater is % price change associated with interest rate change
4. Longer is maturity, more return changes with change in interest rate
5. Bond with high initial interest rate can still have negative return if $i \uparrow$

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Conclusion from Table 2 Analysis

- Prices and returns more volatile for long-term bonds because have higher interest-rate risk
- No interest-rate risk for any bond whose maturity equals holding period

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Exercise 2: Understanding returns!

Question: What is the return on a 5% coupon bond that initially sells for \$1000 and sells for \$960 next year?

Answer:

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INTEREST RATES AND INFLATION

Note: These lecture notes are incomplete without having attended lectures.

Inflation and interest rates

- **Nominal interest rate**, i , is not adjusted for inflation
 - Def: **Real Interest Rate**
 - (Nominal) Interest rate that is adjusted for expected changes in the price level (i.e. expected inflation)
- $$r = i - \pi$$
- Real interest rate more accurately reflects true cost of borrowing

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Returns and Borrowing/Lending

- When real rate is low, greater incentives to borrow and less to lend
 - if $i = 5\%$ and $\pi^e = 3\%$ then:

$$r = 5\% - 3\% = 2\%$$
 - if $i = 8\%$ and $\pi^e = 10\%$ then

$$r = 8\% - 10\% = -2\%$$
- When real rate is high, lower incentive to borrow, but greater incentive to lend

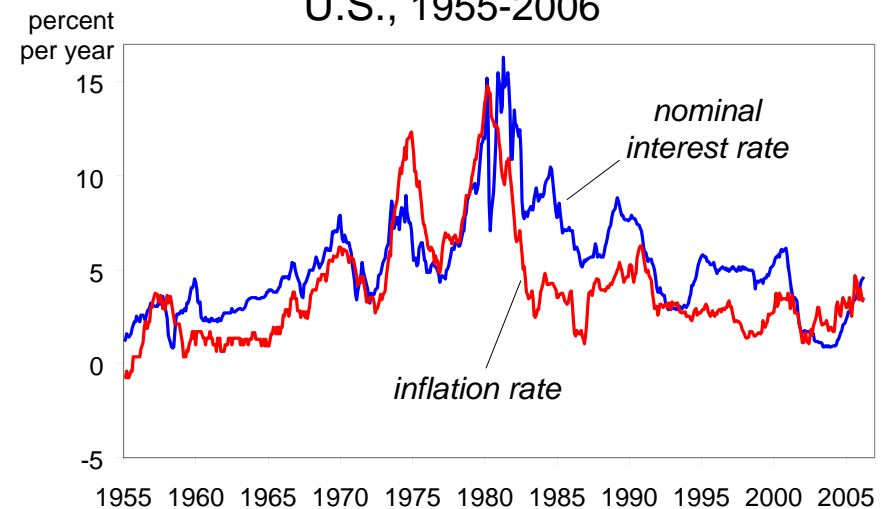
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The Fisher effect

- The Fisher equation: $i = r + \pi$
- Later in the course, we will see that the market for loanable funds: $S = I$ (or equivalently, the market for bonds) determines r .
- Hence, an increase in π causes an equal increase in i .
- This one-for-one relationship is called the **Fisher effect**.

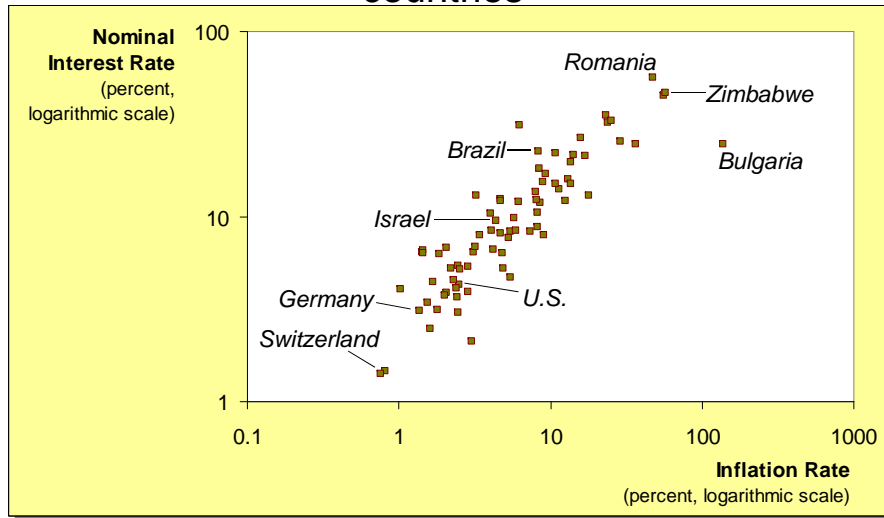
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Inflation and nominal interest rates in the U.S., 1955-2006



Note: These lecture notes are incomplete without having attended lectures.

Inflation and nominal interest rates across countries



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Exercise 3: An Application of the Quantity Theory and the Fisher Effect

Consider the Quantity Theory Equation we studied in the previous lecture: $MV = PY$

Suppose V is constant, M is growing 5% per year, Y is growing 2% per year, and $r = 4$.

- Solve for i .
- If the Fed increases the money growth rate by 2 percentage points per year, find Δi .
- Suppose the growth rate of Y falls to 1% per year.
 - What will happen to π ?
 - What must the Fed do if it wishes to keep π constant?

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Two real interest rates

- π = actual inflation rate
(not known until after it has occurred)
- π^e = expected inflation rate
- $i - \pi^e$ = **ex ante** real interest rate:
the real interest rate people expect
at the time they buy a bond or take out a loan
- $i - \pi$ = **ex post** real interest rate:
the real interest rate actually realized

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Nominal vs. Real Rates

- To grasp the concept: Think about the real return to \$1... [intertemporal price interpretation]
- A saver/lender can use that \$1 to buy $\$1/P_t$ units of goods today
- ... alternatively can save/lend and use the nominal return to buy $\$(1+i)/P_{t+1}$ units of goods tomorrow
- Thus changes in the nominal interest rate account for changes in the percentage changes in prices, i.e. changes in inflation.

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