Intermediate Macroeconomics:
ECON 302 Professor Yamin Ahmad

Problem Set 5: Consumption Theory and Growth Theory

Answer all the parts of the questions separately giving your reasons for your answer.

1. **Consumption Theory**

Consider a 2 period economy where the preferences of a representative household is reflected by the following Cobb-Douglas utility function:

$$U(C_1, C_2) = C_1^{0.5} C_2^{0.5} \tag{1}$$

Suppose that this representative household is expected to get income (in real terms) equal to 100 in the first period, and 126 in the second period. Moreover, assume that the market interest rate is 5% (i.e. $r = 0.05$) and that the representative consumer has perfect foresight.

a. Write down the flow budget constraints.

b. Use your answers to part (a) above to write down the present value budget constraint.

c. Calculate the marginal rate of substitution given preferences depicted in equation (1).

d. Write down the consumption Euler equation and intuitively describe what it states.

e. Solve for the optimal values of consumption in periods 1 and 2 at the perfect foresight equilibrium, i.e. C_1^* and C_2^*.

2. **Neoclassical Theory of Distribution**

Consider the following Cobb-Douglas production function:

$$Y = K^{0.4} L^{0.6} \tag{2}$$

a. Derive the factor demands for capital and labor

b. Derive the factor prices. Show that the rate of return on capital, r, depends negatively to the capital labor ratio (i.e. $\frac{K}{L}$) whilst the real wage rate, w, depends positively with regards to $\frac{K}{L}$.

c. Calculate the shares of capital and labor in output.

3. **Solow Growth Model**

Suppose that an economy has the following characteristics: the marginal propensity to save, $s = 0.25$, the rate of population growth is 3 percent, the rate of depreciation is 2 percent and that the growth rate of exogenous technological progress is 5 percent. Moreover, suppose that the aggregate production function is given by:

$$Y = K^{0.4} (AL)^{0.6} \tag{3}$$

where K represents the stock of capital in the economy; L represents the population and A represents technological progress.

a. Show that the production function in equation (3) above exhibits the properties of a neoclassical production function (i.e. positive and diminishing marginal products).

b. Show that the production function in equation (3) above exhibits constant returns to scale.

c. Re-write the production function above in terms of efficienty units.

d. Derive the equation for capital accumulation using the production function above.

For the questions below, suppose that $A = 1$, and $L = 100$.

e. Calculate the value of the capital stock (in efficieny units) at the steady state to 3 decimal places.

f. Calculate the value of the level of the capital stock (i.e. K, NOT $\tilde{k}$) and the level of output (i.e. Y NOT $\tilde{y}$) at the steady state to the nearest whole number. [Hint: Recall that $\tilde{k} = \frac{K}{AL}$ and $\tilde{y} = \frac{Y}{AL}$].

g. Calculate the value of the real wage and the real rental rate of capital to 3 decimal places.

h. Assuming that this economy is a competitive economy, what would the real rate of interest have to be at in order to maximize consumption at the steady state?