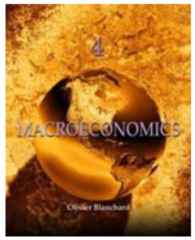
Intermediate Macroeconomics

ECON 302

Professor Yamin Ahmad

- Review of Mathematics



In This Lecture…

Review of Mathematics

- Graphs
- Slopes

Graphing Data

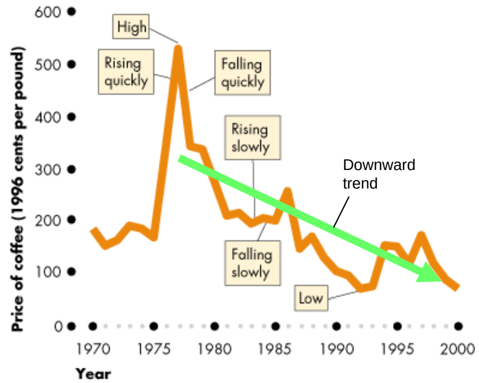
- Economists use three types of graphs to reveal relationships between variables. They are:
  - Time-series graphs
  - Cross-section graphs
  - Scatter diagrams

Graphing Data

Time-Series Graphs

- A **time-series graph** measures time (for example, months or years) along the *x*-axis and the variable or variables in which we are interested along the *y*-axis.
- The time-series graph on the next slide shows the price of coffee between 1970 and 2000.
- The graph shows the level of the price, how it has changed over time, when change was rapid or slow, and whether there was any trend.

Graphing Data

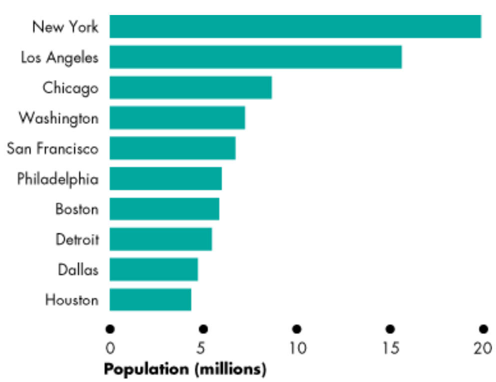


Graphing Data

Cross-Section Graphs

- A **cross-section graph** shows the values of a variable for different groups in a population at a point in time.
- The cross-section graph on the next slide enables you to compare the number of people who live in 10 metropolitan areas in the United States.

Graphing Data

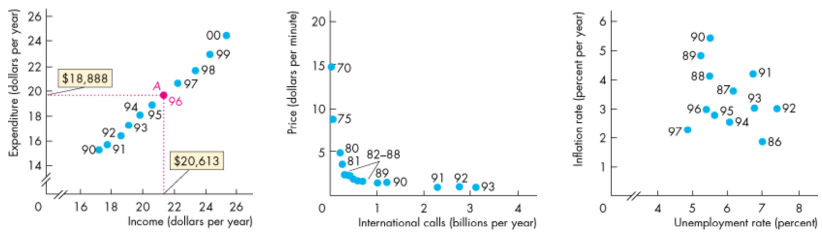


Graphing Data

Scatter Diagrams

- A **scatter diagram** plots the value of one variable on the *x*-axis and the value of another variable on the *y*-axis.
- A scatter diagram can make clear the relationship between two variables.
- The three scatter diagrams on the next slide show examples of variables that move in the same direction, in opposite directions, and in no particular relationship to each other.

Graphing Data



(a) Expenditure and income (b) International phone calls and prices (c) Unemployment and inflation

Graphs Used in Economic Models

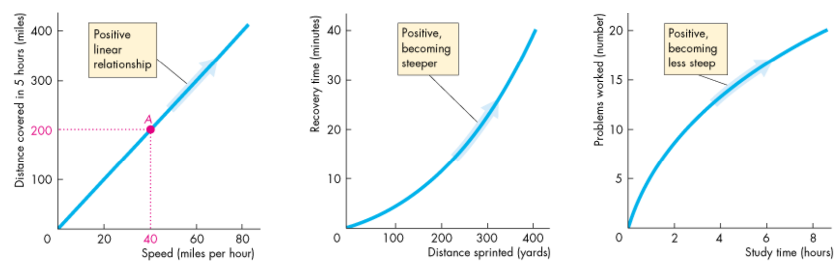
- Graphs are used in economic models to show the relationship between variables.
- The patterns to look for in graphs are the four cases in which:
  - Variables move in the same direction
  - Variables move in opposite directions
  - Variables have a maximum or a minimum (extremals)
  - Variables are unrelated

Graphs Used in Economic Models

Variables That Move in the Same Direction

- A relationship between two variables that move in the same direction is called a **positive relationship** or a **direct relationship**.
- A line that slopes upward shows a positive relationship.
- A relationship shown by a straight line is called a **linear relationship**.
- The three graphs on the next slide show positive relationships.

Graphs Used in Economic Models



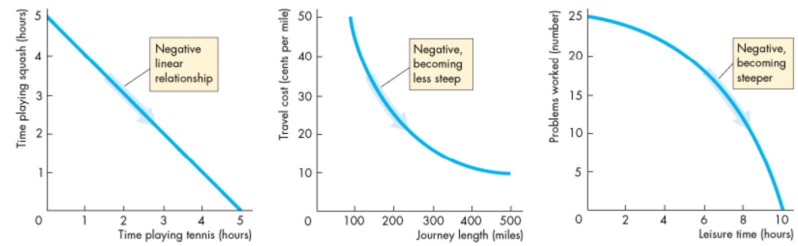
(a) Positive linear relationship (b) Positive, becoming steeper (c) Positive, becoming less steep

Graphs Used in Economic Models

Variables That Move in Opposite Directions

- A relationship between two variables that move in opposite directions is called a **negative relationship** or an **inverse relationship**.
- A line that slopes downward shows a negative relationship.
- The three graphs on the next slide show negative relationships.

Graphs Used in Economic Models



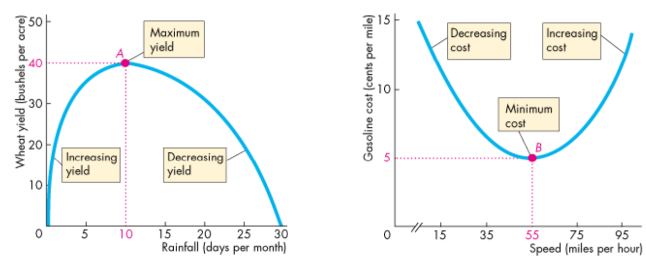
(a) Negative linear relationship (b) Negative, becoming less steep (c) Negative, becoming steeper

Graphs Used in Economic Models

Extremals: Variables That Have a Maximum or a Minimum

- The two graphs on the next slide show relationships that have a maximum and a minimum.
- These relationships are positive over part of their range and negative over the other part.

Graphs Used in Economic Models



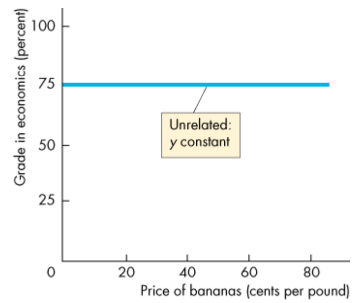
(a) Relationship with a maximum (b) Relationship with a minimum

Graphs Used in Economic Models

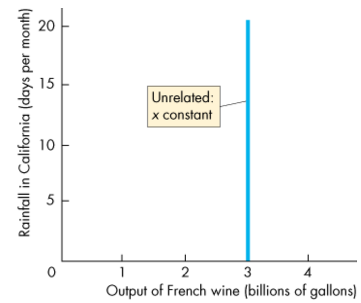
Variables That are Unrelated

- Sometimes we want to emphasize that two variables are unrelated.
- The two graphs on the next slide show examples of variables that are unrelated.

Graphs Used in Economic Models



(a) Unrelated: *y* constant



(b) Unrelated: *x* constant

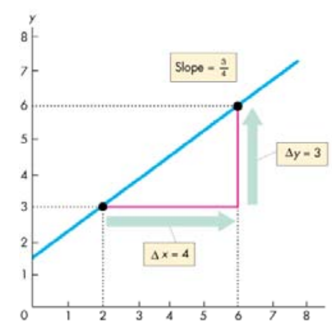
The Slope of a Relationship

- The **slope** of a relationship is the change in the value of the variable measured on the *y*-axis divided by the change in the value of the variable measured on the *x*-axis.
- We use the Greek letter Δ (capital delta) to represent "change in."
- So Δy means the change in the value of the variable measured on the *y*-axis and Δx means the change in the value of the variable measured on the *x*-axis.
- The slope of the relationship is $\Delta y/\Delta x$.

The Slope of a Relationship

The Slope of a Straight Line

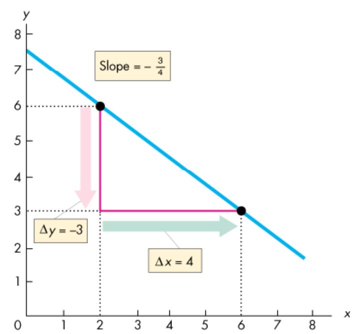
- The slope of a straight line is constant.
- Graphically, the slope is calculated as the "rise" over the "run."
- The slope is positive if the line is upward sloping.



(a) Positive slope

The Slope of a Relationship

- The slope is negative if the line is downward sloping.



(b) Negative slope

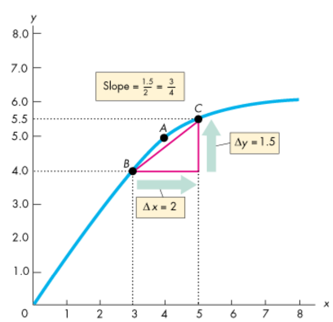
The Slope of a Relationship

- The Slope of a Curved Line
  - ➢ The slope of a curved line at a point varies depending on where along the curve it is calculated.
  - ➢ We can calculate the slope of a curved line either at a point or across an arc.

The Slope of a Relationship

Slope Across an Arc

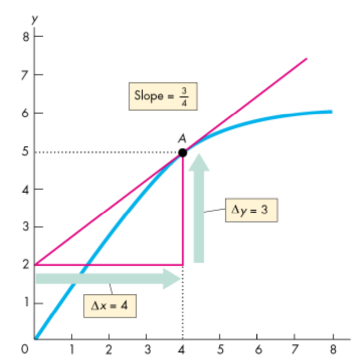
- The *average* slope of a curved line across an arc is equal to the slope of a straight line that joins the endpoints of the arc.
- Here, we calculate the average slope of the curve along the arc *BC*.



The Slope of a Relationship

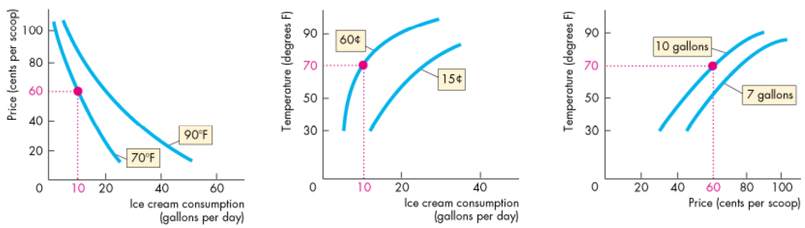
Slope at a Point

- The slope of a curved line at a point is equal to the slope of a straight line that is the tangent to that point.
- Here, we calculate the slope of the curve at point *A*.



Graphing Relationships Among More Than Two Variables

- When a relationship involves more than two variables, we can plot the relationship between two of the variables by holding other variables constant—by using *ceteris paribus*.
- Here we plot the relationships among three variables.



(a) Price and consumption at a given temperature
(b) Temperature and consumption at a given price
(c) Temperature and price at a given consumption

Mathematics: What You Should Be Able To Do

- Graphs are a way to do math. But sometimes it is too hard to represent models with graphs. Then we use math.
- This moves us closer to what professional economists do:-
  - Algebra: Manipulate equations and solve for endogenous variables in terms of exogenous variables
  - Calculate the inverse of functions
  - Calculate the effects of changes in variables on other variables (- essentially working out slopes!)
  - Calculate percentage changes (- useful for calculating growth rates)
  - Examine correlations between variables

More on Calculating Slopes

Slope of a Linear Function

- Consider a linear function:

$$y = f(x) = mx + c$$

- Suppose that the pairs (x_0,y_0) and (x_1,y_1) both represent arbitrary points on the line. Then the ratio:

$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$

  is called the slope.
- c is called the intercept
- Question: What is the slope of a line that goes through the points (4,6) and (0,7)?

Solution…

- Take the points (4,6) and (0,7)
- If we plug into the formula on the previous page:

$$\frac{\Delta y}{\Delta x} = m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$= \frac{7-6}{0-4} = \frac{1}{(-4)} = -\frac{1}{4}$$

- Hence the slope of a line that goes through the points (4,6) and (0,7) is -1/4.

Another Practice Problem… (Solution next page)

Centigrade and Fahrenheit

- Let C denote the temperature in degrees Centigrade and let F denote the temperature in degrees Fahrenheit.
- We know that 0° Centigrade is 32° Fahrenheit is the freezing temperature of water, and that 100° Centigrade or 212° Fahrenheit is the boiling point for water.
- What is the equation that relates degrees Fahrenheit to degrees Centigrade? What is the inverse function, i.e. the equation that relates degrees Centigrade to degrees Fahrenheit?

Practice Problem… Answer

Centigrade and Fahrenheit

- To find the equation of the line through points (0,32) and (100,212), we first find the slope:

$$slope, m = \frac{\Delta F}{\Delta C} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

- This means an increase of 1° Centigrade corresponds to an increase of 9/5° Fahrenheit. Hence we can use the slope, 9/5 and the point (0,32) to express the linear relationship:

$$\frac{F-32}{C-0} = \frac{9}{5} \quad \text{or} \quad F = \frac{9}{5}C + 32$$

- Or equivalently (re-arranging for *C=f(F)*)

$$9C = 5(F-32) \quad i.e. \quad C = \frac{5}{9}(F-32)$$

Application to Economics…

Recall From Lecture 1…

- Models embody assumptions about individual behavior, market structure and what is exogenous (including policy regime).
- Solution gives us the endogenous variables in terms of the exogenous factors.

Mathematical functions

- We use functional notation when we want to express the idea that one variable is determined by other variables.
  - For example, supply of pizzas is a function of the price of pizzas and the price of materials (price of inputs):

$$Q^s = S(P, P_m)$$

- In this example, the quantity supplied of pizza is the "endogenous" variable in the pizza supply model.
  - The price of pizza and the price of materials are "exogenous" for the pizza maker. She cannot influence those prices (assuming that she is not a monopoly seller of pizza!)

More on Endogenous and Exogenous variables

- Variables that are exogenous in some models might be endogenous in other models.
  - For example, in many macro models, we might take the share of income earned by females vs. males as exogenous.
  - But some economic models are designed exactly to explain those shares.
- In some cases, a variable is exogenous in the building block of a more general model, but endogenous in the general model.
  - Price of pizza is exogenous for the pizza supplier, but determined within our model of the pizza market.

The Pizza market

- We can take the equations for supply of pizza, demand for pizza, and market equilibrium:

$$Q^s = S(P, P_m)$$

$$Q^d = D(P, Y)$$

$$Q^s = Q^d$$

- P and P_m are exogenous for the pizza supplier. P and Y are exogenous for the pizza demander.
- These three equations together determine Q^s, Q^d, and P endogenously. The exogenous variables for the pizza market are P_m and Y.

Macroeconomic example

- We will model aggregate consumption as depending on "disposable" income:

$$C = C(Y - T)$$

  - For consumers in this model, income and taxes are exogenous.
  - But aggregate income will be determined in our macro model.
- In most of our macro models, aggregate taxes will be exogenous, but sometimes they will be endogenous.
- In most of our models of consumer's income is exogenous, but sometimes it is endogenous. Income depends on how many hours we work, for example.

Algebra

- Review how to solve endogenous variables.
- Task: Solve for P and Q, in terms of P_m and Y:

$$Q = a + bP - cP_m \qquad Q = d - eP + fY$$

  - *a, b, c, d, e,* and *f* are "parameters". For example, *e* tells us how much demand falls when the price rises.

Solution

- Here is the solution:

$$Q = \frac{bd + ae}{b + e} - \frac{ce}{b + e}P_m + \frac{bf}{b + e}Y$$

$$P = \frac{d - a}{b + e} + \frac{c}{b + e}P_m + \frac{f}{b + e}Y$$

- A lot of information is embedded in these solutions:
  - e.g. amongst other things, we can see here how the sensitivity of supply to output prices, b, or the sensitivity of demand to income, f, influences equilibrium prices and quantity.