



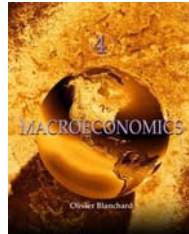
# Intermediate Macroeconomics

ECON 302

Professor Yamin Ahmad

## Lecture 2:

- Review of Mathematical Concepts
- Business Cycle Facts



## In This Lecture...

- Review of Mathematical Concepts
  - Slopes
  - Basic Calculus

## Business Cycle Facts

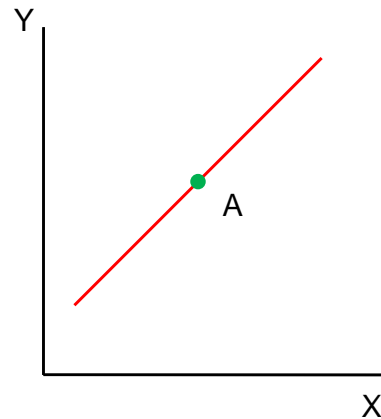
- Co-movements of variables with output gap
- Cyclical behavior
  - Procyclical
  - Countercyclical
  - Acyclical

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## Movements versus Shifts of Curves

- What causes a movement along a graph, what causes a curve to shift?
- The easiest way to think about it is for a straight line (linear) graph
- Changes in Y or X causes a point like A to move along the curve

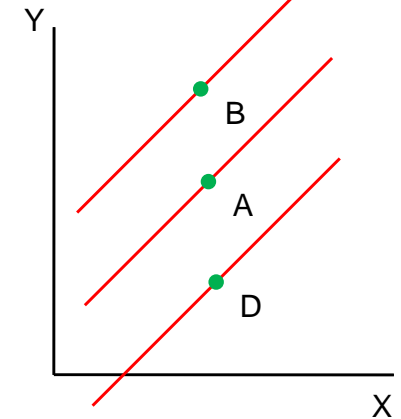


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## Movements versus Shifts of Curves

- Anything else that affects the relationship between Y and X, e.g. "c" will shift the curve
- Suppose that there is a positive relationship between "c" and Y, then an increase in c will shift the curve up from A to a point like B
- If a negative relationship exists between "c" and Y, an increase in c shifts the curve downwards from A to a point like D



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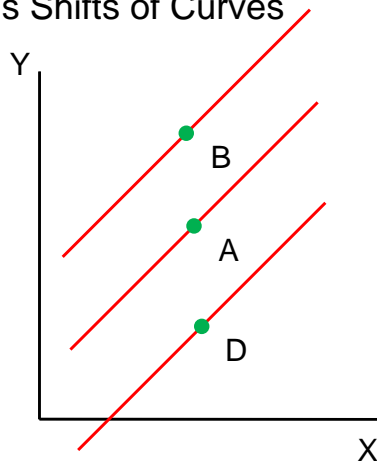
## Movements versus Shifts of Curves

- Algebraically, for a straight line:

$$Y = mX + c$$

where

- Y is the **dependent variable** (the thing we are looking to explain)
- X is the **independent variable** (the thing we are using to explain/ relating to Y)
- m is the **slope**
- c is the **intercept**
- Thus anything else that impacts Y (apart from X) will change the intercept, c, shifting the curve!



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## The Slope of a Relationship

- The **slope** of a relationship is the change in the value of the variable measured on the y-axis divided by the change in the value of the variable measured on the x-axis.
- We use the Greek letter  $\Delta$  (capital delta) to represent “change in.”
- So  $\Delta y$  means the change in the value of the variable measured on the y-axis and  $\Delta x$  means the change in the value of the variable measured on the x-axis.
- The slope of the relationship is  $\Delta y/\Delta x$ .

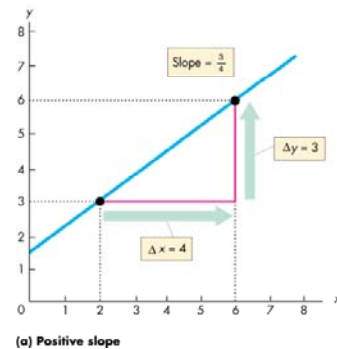
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## The Slope of a Relationship

### The Slope of a Straight Line

- The slope of a straight line is constant.
- Graphically, the slope is calculated as the “rise” over the “run.”
- The slope is positive if the line is upward sloping.



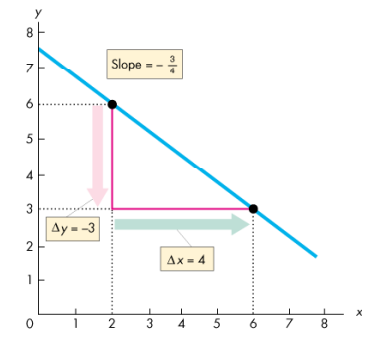
(a) Positive slope

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## The Slope of a Relationship

- The slope is negative if the line is downward sloping.



(b) Negative slope

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## The Slope of a Relationship

### The Slope of a Curved Line

- The slope of a curved line at a point varies depending on where along the curve it is calculated.
- We can calculate the slope of a curved line either at a point or across an arc.

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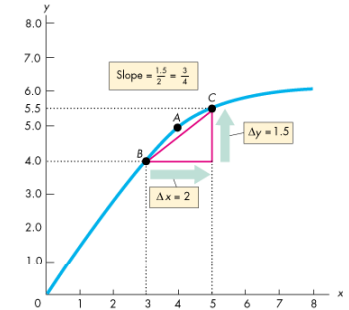
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## The Slope of a Relationship

### Slope Across an Arc

- The *average* slope of a curved line across an arc is equal to the slope of a straight line that joins the endpoints of the arc.
- Here, we calculate the average slope of the curve along the arc *BC*.



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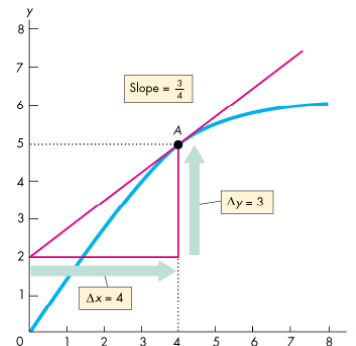
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## The Slope of a Relationship

### Slope at a Point

- The slope of a curved line at a point is equal to the slope of a straight line that is the tangent to that point.
- Here, we calculate the slope of the curve at point *A*.



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## More on Calculating Slopes

### Slope of a Linear Function

- Consider a linear function:  
$$y = f(x) = mx + c$$
- Suppose that the pairs  $(x_0, y_0)$  and  $(x_1, y_1)$  both represent arbitrary points on the line. Then the ratio:  
$$m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$
 is called the slope.
- $c$  is called the intercept
- Question: What is the slope of a line that goes through the points  $(4,6)$  and  $(0,7)$ ?

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### Solution...

- Take the points (4,6) and (0,7)
- If we plug into the formula on the previous page:

$$\frac{\Delta y}{\Delta x} = m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$= \frac{7 - 6}{0 - 4} = \frac{1}{(-4)} = -\frac{1}{4}$$

- Hence the slope of a line that goes through the points (4,6) and (0,7) is -1/4.



### Another Practice Problem...

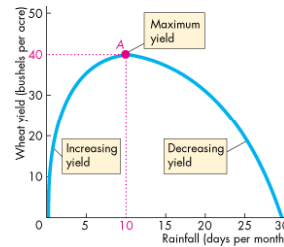
#### Centigrade and Fahrenheit

- Let C denote the temperature in degrees Centigrade and let F denote the temperature in degrees Fahrenheit.
- We know that 0° Centigrade is 32° Fahrenheit is the freezing temperature of water, and that 100° Centigrade or 212° Fahrenheit is the boiling point for water.
- What is the equation that relates degrees Fahrenheit to degrees Centigrade? What is the inverse function, i.e. the equation that relates degrees Centigrade to degrees Fahrenheit?

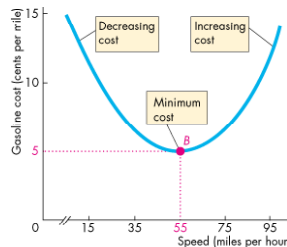


### How to find Extremals ...?

- Consider the two graphs on the right...
- ... how would we find any maxima or minima that exist?
- Answer:
  - Look for “stationary points”, i.e. where the slopes of the curves are zero!



(a) Relationship with a maximum



(b) Relationship with a minimum



### How to find Extremals ...?

- To find any maxima or minima, use the following methodology:
  1. Calculate the slope of the function  $y=f(x)$ , i.e.  $\Delta y/\Delta x$
  2. Set  $\Delta y/\Delta x=0$  and find values of x where the slope equals zero!
  3. Check around those points found in step 2.
    1. If the slope is increasing then decreasing, it is a maximum
    2. If the slope is decreasing then increasing, then it is a minimum.

## Slope Terminology

- For a linear function:  $y = f(x) = mx + c$ , the slope is easy to calculate:

$$\text{slope} = \frac{\Delta y}{\Delta x} = m$$

- When the change,  $\Delta$ , is very (very, very, very, ...) small, i.e. as  $\Delta \rightarrow 0$ , we write the change in  $y$  relative to  $x$  as:

$$\frac{dy}{dx}$$

and call it the “*derivative*” of  $y$  with respect to  $x$

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## Some Rules to Calculate Slopes

However, for **nonlinear functions**, use the following rules:

- The slope of a constant = 0. e.g. slope of  $y=5$  is 0.
- The slope of a variable to the  $k^{\text{th}}$  power, e.g.  $y=x^k$  is:

$$\frac{\Delta y}{\Delta x} = \frac{dy}{dx} = kx^{k-1},$$

For example: the slope of  $y=x^4$  is:  $\frac{dy}{dx} = 4x^3$

- $y=Ax^k$  is  $\frac{dy}{dx} = Akx^{k-1}$ , for example: for  $y=5x^4$ , the slope is:

$$\frac{\Delta y}{\Delta x} = 5 * 4x^3 = 20x^3$$

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## Some Rules to Calculate Slopes (cont.)

The slope of:

- Combining (3) and (1), we get:

- For  $y = Ax^k + c$ ,  $\frac{\Delta y}{\Delta x} = Akx^{k-1}$ ,

e.g. for  $y=5x^4 + 10$ ,  $\frac{\Delta y}{\Delta x} = 5 * 4x^3 = 20x^3$

- For  $y = Ax^k + Bx^n$ ,  $\frac{\Delta y}{\Delta x} = Akx^{k-1} + Bnx^{n-1}$ , e.g. for  $y=5x^4 + 3x^5$

$$\frac{\Delta y}{\Delta x} = 5 * 4x^3 + 3 * 5x^4 = 20x^3 + 15x^4$$

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## Some Rules to Calculate Slopes (cont.)

- If  $y$  is a function of several variables, e.g.  $y=f(x,s,t)$ , treat the other variables as constant when calculating the slope of  $y$  wrt  $x$ ! e.g.  $y=100 + 5x^4 + 10s^2 + 15t^3$

$$\frac{dy}{dx} = 5 * 4x^3 = 20x^3;$$

Similarly  $\frac{dy}{ds} = 10 * 2s = 20s;$

$$\frac{dy}{dt} = 15 * 3t = 45t$$

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### Practice Questions

Calculate the slopes for the following functions:

1.  $y = x^3 - 12x^2 + 36x + 8$

2.  $y = x^2 + 2xt + t^2$

3.  $y = x^2t + t^2x - 2xt + 5x - 3t$

4.  $y = 10k^\alpha L^{(1-\alpha)}$

5.  $y = 5\ln x + 10^x$

Quick Summary :

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$x^m$	$mx^{m-1}$
$Ae^{bx}$	$Abe^{bx}$
$\ln x$	$(1/x)$
$a^x$	$a^x \ln a$



### The Business Cycle

What is a **Business Cycle**?

- The business cycle is the periodic but irregular up-and-down movement in production and jobs.



### The Business Cycle

- The NBER defines the phases and turning points of the business cycle as follows:
  - A **recession** is a significant decline in activity spread across the economy, lasting more than a few months, visible in industrial production, employment, real income, and wholesale-retail trade.
  - A recession begins just after the economy reaches a **peak** of activity and ends as the economy reaches its **trough**. Between trough and peak, the economy is in an **expansion**.



### Economic Growth and Fluctuations

Every business cycle has **two phases**:

1. A recession
2. An expansion

and **two turning points**:

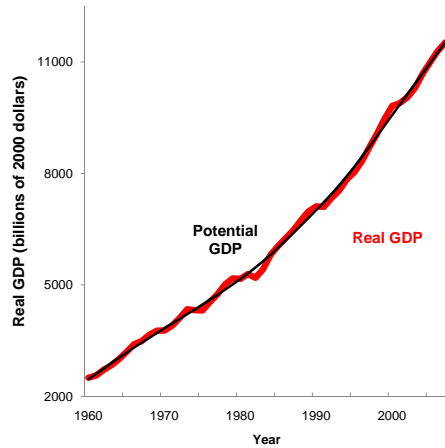
1. A peak
2. A trough

- A **recession** is a period during which real GDP decreases for at least two successive quarters.
- An **expansion** is a period during which real GDP increases.



## Economic Growth and Fluctuations

- Economic Growth in the United States
  - Figure 1 on the right shows real GDP in the United States from 1962 to 2007.
- The figure highlights:
  - Fluctuations of real GDP
  - Smoother growth of potential GDP



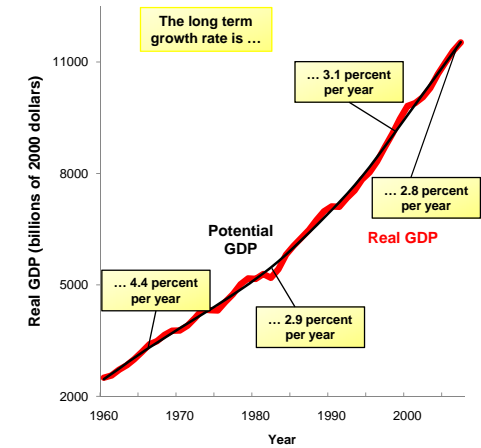
Source: Bureau of Economic Analysis

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## Economic Growth and Fluctuations

- **Potential GDP** is the value of real GDP when all the economy's labour, capital, land, and entrepreneurial ability are fully employed.
- During the 1970s and early 1980s, real GDP growth slowed—a **productivity growth slowdown**.



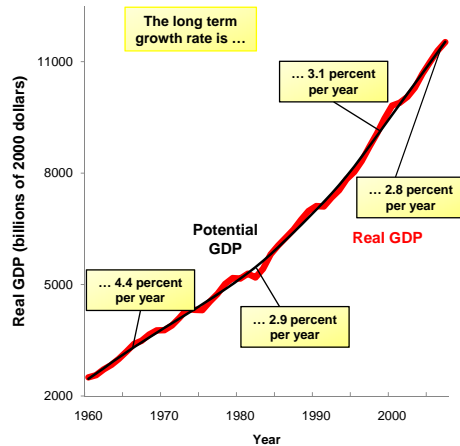
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## Economic Growth and Fluctuations

- Real GDP fluctuates around potential GDP in a **business cycle**
  - a periodic but irregular up-and-down movement in production.

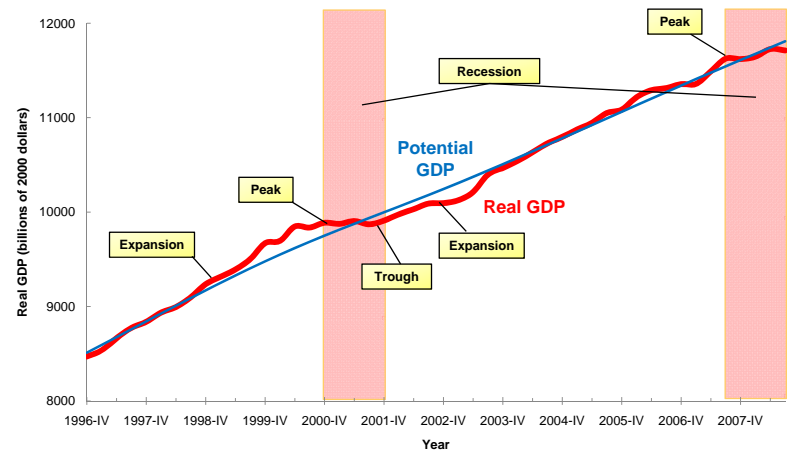


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## Economic Growth and Fluctuations

- This figure shows the most recent U.S. cycles.

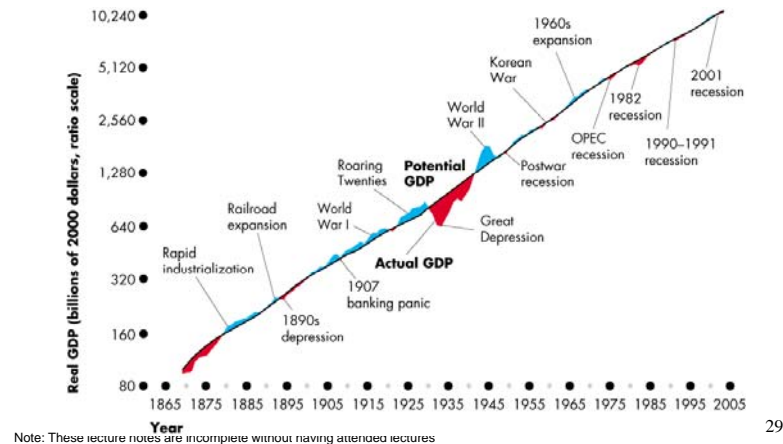


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## Economic Growth and Fluctuations

- This figure shows the long-term growth trend and cycles



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## Production (Real GDP) as a Benchmark

- In Macroeconomics, we compare what happens to different variables in terms of how it relates to production in the economy (i.e. how does inflation, or unemployment relate to real GDP?)
- Definition:
  - **Procylical**: the variable moves with the business cycle (i.e. it *increases* when production *increases* and vice versa)
  - **Countercyclical**: the variable moves in the opposite direction of the business cycle (i.e. it *increases* when production *decreases* and vice versa)
  - **Acyclical**: does not move with the business cycle

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## Thinking About Cyclicity...

- As a general rule of thumb:
  - A variable, X, is **procyclical** if  $\text{Correlation}(X, \text{output gap})$  or  $\text{Correlation}(X, \text{economic growth rate}) > 0.25$
  - A variable, X, is **countercyclical** if  $\text{Correlation}(X, \text{output gap})$  or  $\text{Correlation}(X, \text{economic growth rate}) < -0.25$
  - A variable, X, is **acyclical** if  $-0.25 < \text{Correlation}(X, \text{output gap}) < 0.25$
- In general the closer the correlation is to +1 ( -1) (i.e. the further away it is from zero), the greater the degree of pro-cyclicity (counter-cyclicity).

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## Some Facts about the Business Cycle

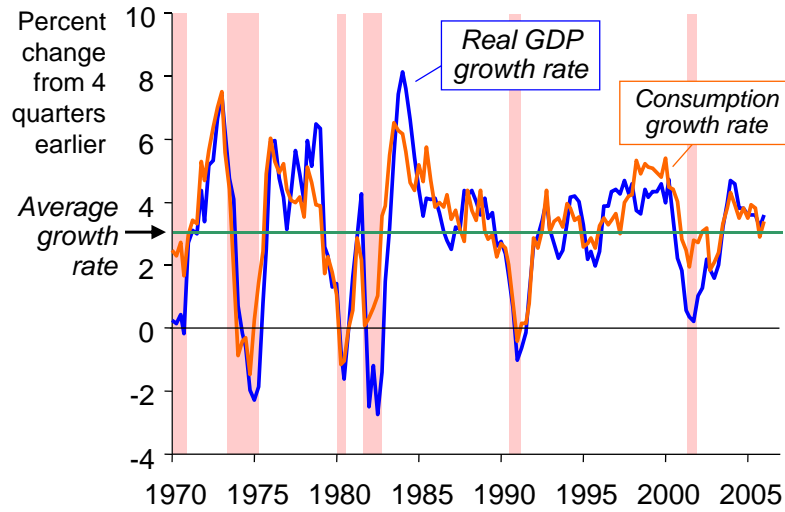
- GDP growth averages 3–3.5 percent per year over the long run with large fluctuations in the short run.
- Consumption and investment fluctuate with GDP, but consumption tends to be less volatile and investment more volatile than GDP.
  - Correlation of consumption and the output gap: 0.998
  - Correlation of investment and the output gap: 0.989
- Unemployment rises during recessions and falls during expansions.
  - Correlation of unemployment and GDP is approx -0.67
- Okun's Law**: the negative relationship between GDP and unemployment.

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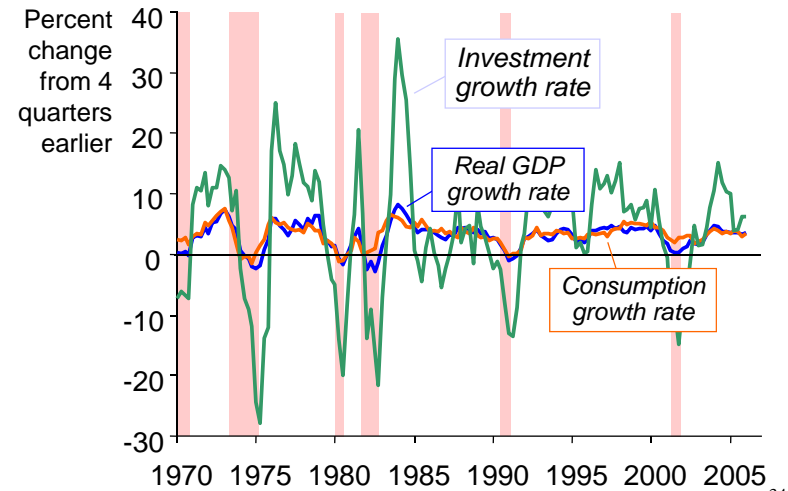
### Growth rates of Real GDP, Consumption



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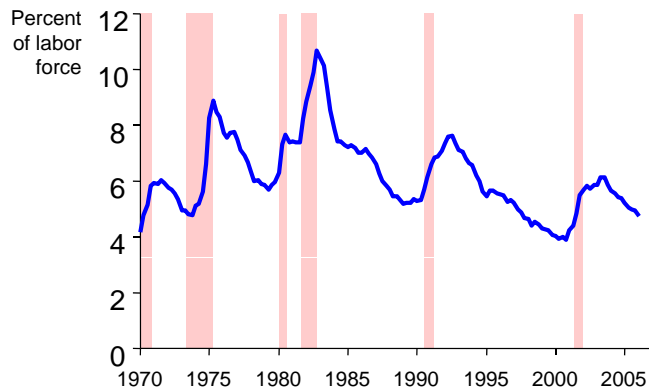
### Growth rates of Real GDP, Consumption, Investment



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### Unemployment

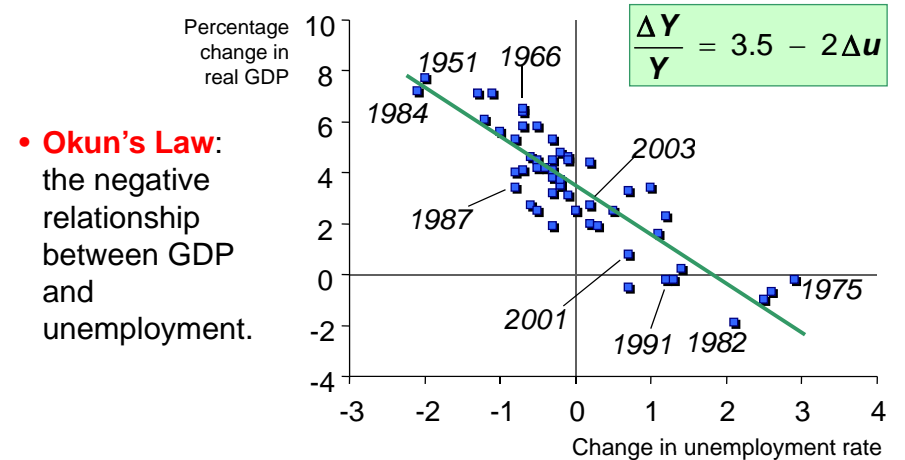


- Unemployment rises in recessions and falls in expansions
- Unemployment rate is a lagging indicator

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### Okun's Law



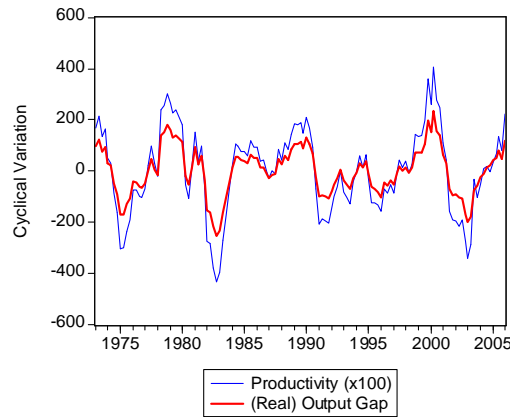
- **Okun's Law:** the negative relationship between GDP and unemployment.

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## Output Gap and Productivity

- The correlation between productivity and real GDP is approximately 0.9996
- This indicates that productivity is highly procyclical



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## Business Cycle Facts (Quantities)

1. Output movements persist (but not periodic)
2. Industries move together
3. Consumption, Investment and Imports are procyclical
  - they move with the business cycle and this is highlighted by positive correlations
4. Unemployment is countercyclical, but productivity is procyclical

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## Business Cycle Facts (Prices)

5. Prices/Inflation is procyclical.
6. Real wages are acyclical or mildly procyclical.
7. Short term interest rates are procyclical.
8. Money and velocity are procyclical.

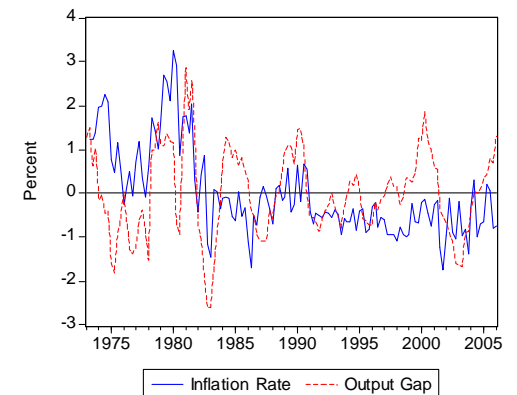
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## Output Gap and Inflation

- The correlation between the CPI and GDP is 0.979.
- The correlation between inflation and GDP is approximately 0.301.
- This indicates that prices are highly procyclical, whilst inflation is mildly procyclical



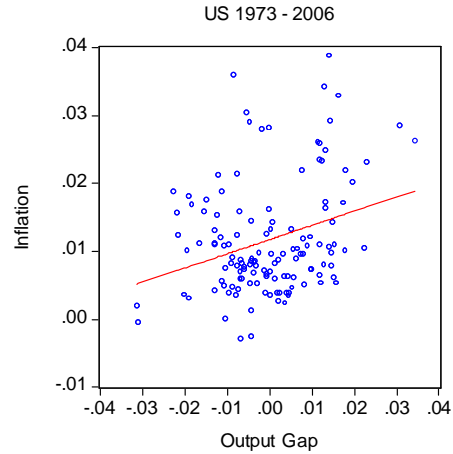
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### Inflation v. Output Gap

- This scatter plot highlights the positive relationship observed between inflation and the output gap.
- The bigger output is (above potential), the greater the amount of inflation!

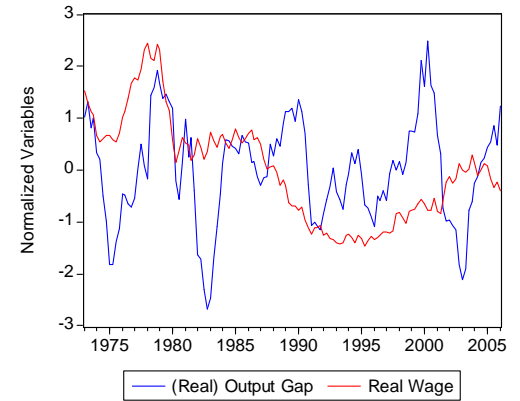


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### Output Gap and Real Wage

- The correlation between real wages and real GDP is approximately 0.124
- This indicates that real wages are acyclical (– i.e. that they are independent of the business cycle) or at best, very mildly procyclical.

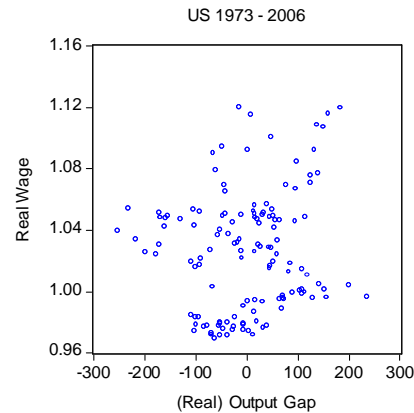


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### Real Wage v. Output Gap

- This scatter plot highlights the lack of a relationship between real wages and the output gap.
- It is not possible to figure out which direction the line of best fit should go!

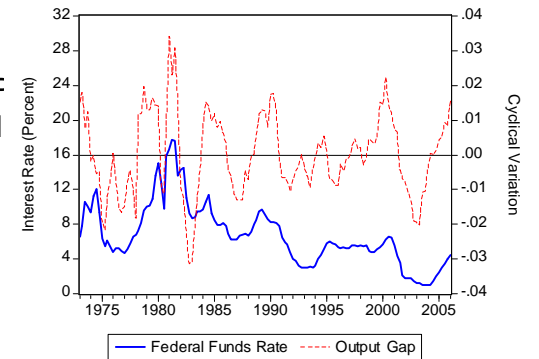


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### Output Gap and Short Interest Rate

- The correlation between short term interest rates and GDP is approximately 0.401
- This indicates that nominal interest rates are procyclical

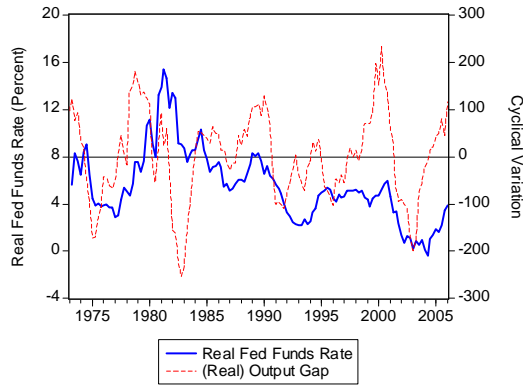


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## Output Gap and Real Short Interest Rate

- The correlation between short term interest rates and Real GDP is approximately 0.384
- This indicates that real interest rates are (mildly) procyclical



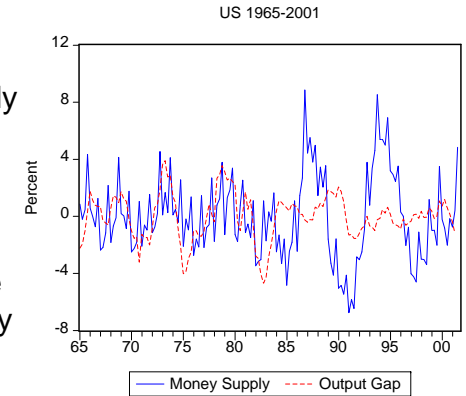
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## Output Gap and Money Supply

- The correlation between money supply and GDP is approximately 0.967
- This indicates that the money supply is highly procyclical



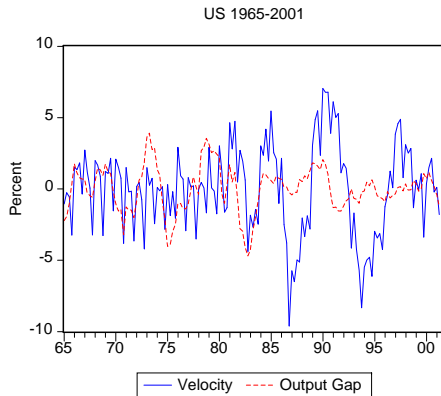
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## Output Gap and Velocity

- The correlation between velocity and GDP is approximately 0.826
- This indicates that velocity is highly procyclical



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