Intermediate Macroeconomics

ECON 302

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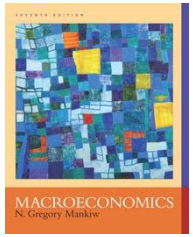
Lecture 13:

- National Income in the Long Run
- Neoclassical Theory of Distribution
- Components of output in the long run



Key Concepts in this Lecture

- Neoclassical Theory of Distribution
- What determines demand for factors
- What determines incomes from factors of production
- Distribution of output in the long run

Note: These lecture notes are incomplete without having attended lectures

National Income In The Long Run

Questions:

- What determines output in the Long Run?
- What determines the incomes of factors of production?
- How is output distributed between the alternative uses?

Note: These lecture notes are incomplete without having attended lectures

Factors of production

K = capital:
tools, machines, and structures used in production

L = labor:
the physical and mental efforts of workers

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The production function: Y = F(K,L)

- shows how much output (***Y***) the economy can produce from ***K*** units of capital and ***L*** units of labor
- reflects the economy's level of technology
- exhibits constant returns to scale

Determination of Output

The production function tells us how much firms produce, given inputs

$$Y = F(K, L)$$

- Positive Marginal Products

$$\frac{\partial Y}{\partial K} \equiv F_K > 0; \qquad \frac{\partial Y}{\partial L} \equiv F_L > 0$$

- Diminishing Marginal Products

$$\frac{\partial^2 Y}{\partial K^2} \equiv F_{KK} < 0; \qquad \frac{\partial^2 Y}{\partial L^2} \equiv F_{LL} < 0$$

Returns to scale: A review

Initially $Y_1 = F(K_1, L_1)$

Scale all inputs by the same factor ***z***:

$K_2 = zK_1$ and $L_2 = zL_1$

(*e.g.*, if $z = 1.2$, then all inputs are increased by 20%)

What happens to output, $Y_2 = F(K_2, L_2)$?

- If **constant returns to scale**, $Y_2 = zY_1$
- If **increasing returns to scale**, $Y_2 > zY_1$
- If **decreasing returns to scale**, $Y_2 < zY_1$

Returns to scale: Example 1

$$F(K,L) = \sqrt{KL}$$

$$F(zK,zL) = \sqrt{(zK)(zL)}$$

$$= \sqrt{z^2KL}$$

$$= \sqrt{z^2}\sqrt{KL}$$

$$= zF(K,L)$$

constant returns to scale for any $z > 0$

Returns to scale: Example 2

$$F(K,L) = \sqrt{K} + \sqrt{L}$$

$$F(zK,zL) = \sqrt{zK} + \sqrt{zL}$$

$$= \sqrt{z}\sqrt{K} + \sqrt{z}\sqrt{L}$$

$$= \sqrt{z}\left(\sqrt{K} + \sqrt{L}\right)$$

$$= \sqrt{z}\,F(K,L)$$

decreasing returns to scale for any $z > 1$

Returns to scale: Example 3

$$F(K,L) = K^2 + L^2$$

$$F(zK,zL) = (zK)^2 + (zL)^2$$

$$= z^2\left(K^2 + L^2\right)$$

$$= z^2 F(K,L)$$

increasing returns to scale for any $z > 1$

NOW YOU TRY:

Returns to Scale

- Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:

(a) $F(K,L) = \dfrac{K^2}{L}$

(b) $F(K,L) = K + L$

Constant Returns to Scale

- Since Constant Returns to Scale means:

$$F\left(zK,zL\right) = zY \qquad z>0$$

- Implications:
  - Complementary Factors

$$F_{KL} > 0$$

  - Euler's Theorem: $Y = KF_K + LF_L$

$$= K\frac{\partial F}{\partial K} + L\frac{\partial F}{\partial L}$$

Assumptions

To start off, assume that in the long run:

1. Technology is fixed.
2. The economy's supplies of capital and labor are fixed at

$$\boldsymbol{K} = \overline{\boldsymbol{K}} \quad \text{and} \quad \boldsymbol{L} = \overline{\boldsymbol{L}}$$

Example of a Production Function

- Most popular production function: **Cobb-Douglas**

$$Y = AK^{\alpha}L^{1-\alpha} \qquad 0 < \alpha < 1$$

A is a constant, "Technology"

- No technological progress or growth
- In the Long Run

$$\overline{Y} = F(\overline{K}, \overline{L})$$

The distribution of national income

- determined by **factor prices**,

the prices per unit firms pay for the factors of production

- **wage rate** = price of ***L***
- **rental rate** = price of ***K***

Notation

W	= nominal wage
R	= nominal rental rate
P	= price of output
W / ***P***	= real wage (measured in units of output)
R / ***P***	= real rental rate

How factor prices are determined

- Factor prices are determined by supply and demand in factor markets.
- Recall: Supply of each factor is fixed.
- What about demand?

Demand for labor

- Assume markets are competitive: each firm takes ***W***, ***R***, and ***P*** as given.
- Basic idea:
  A firm hires each unit of labor if the cost does not exceed the benefit.
  - cost = real wage
  - benefit = marginal product of labor

Marginal product of labor (*MPL*)

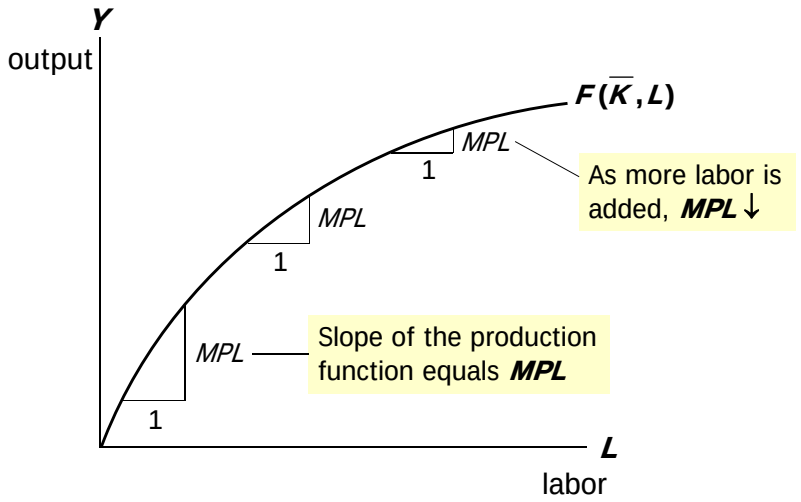
- Definition:
  The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

$$MPL = \frac{\partial F}{\partial L}$$

(or in 202 notation:)

$$\boldsymbol{MPL} = \boldsymbol{F}(\boldsymbol{K}, \boldsymbol{L}+1) - \boldsymbol{F}(\boldsymbol{K}, \boldsymbol{L})$$

MPL and the production function



Diminishing marginal returns

- As a factor input is increased, its marginal product falls (other things equal).

- Intuition:
  Suppose ↑***L*** while holding ***K*** fixed
  ⇒ fewer machines per worker
  ⇒ lower worker productivity

NOW YOU TRY:

Identifying Diminishing Marginal Returns

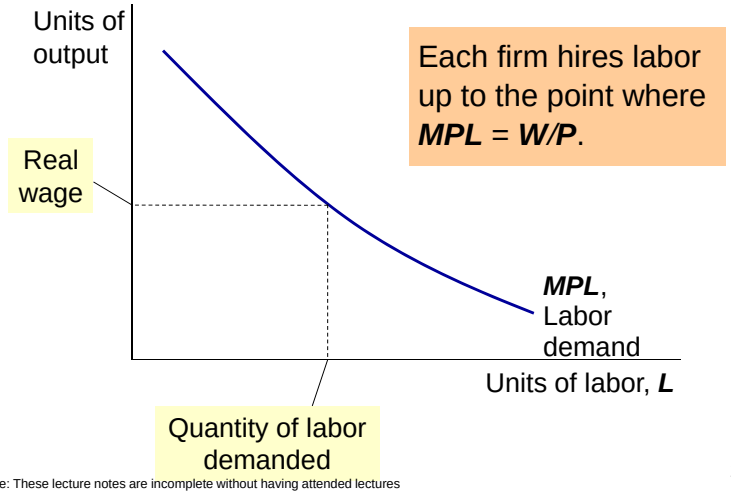
- Which of these production functions have diminishing marginal returns to labor?

  a) $F(K,L) = 2K + 15L$

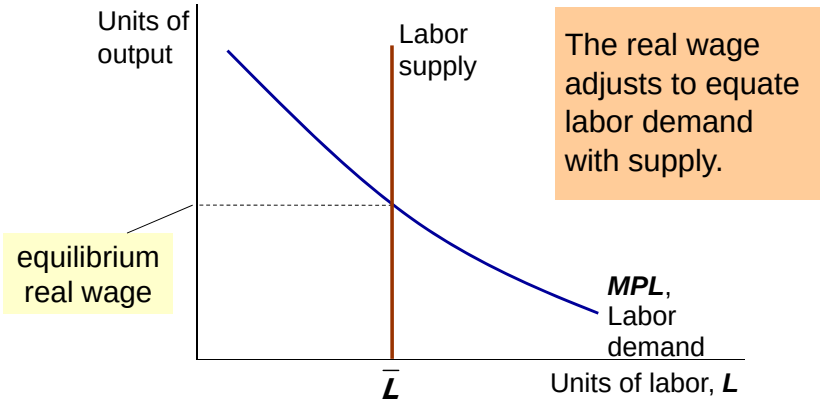
  b) $F(K,L) = \sqrt{KL}$

  c) $F(K,L) = 2\sqrt{K} + 15\sqrt{L}$

MPL and the demand for labor



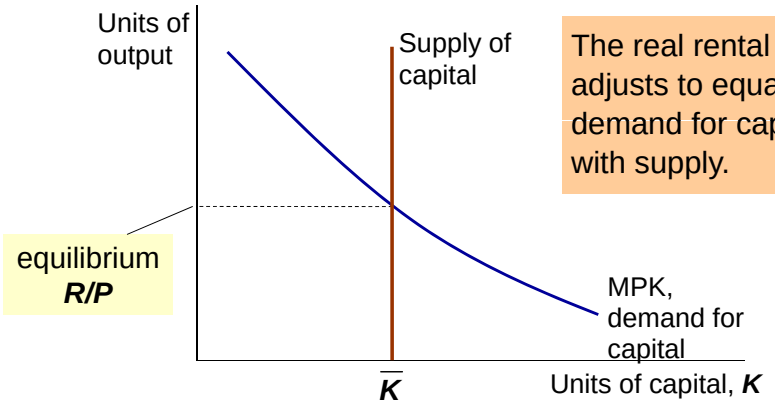
The equilibrium real wage



Determining the rental rate

- We have just seen that ***MPL*** = ***W***/***P***.
- The same logic shows that ***MPK*** = ***R***/***P***:
  - diminishing returns to capital: ***MPK*** ↓ as ***K*** ↑
  - The ***MPK*** curve is the firm's demand curve for renting capital.
  - Firms maximize profits by choosing ***K*** such that ***MPK*** = ***R***/***P***.

The equilibrium real rental rate



The real rental rate adjusts to equate demand for capital with supply.

Neoclassical Theory of Distribution

- The theory that gives us factor incomes is called the "Neoclassical Theory of Distribution".
- Methodology:
  - First derive demand for factors, then equate to supply. This gives us an equilibrium price.
  - Derive equilibrium level of wages and interest rate (rate of return to capital)

$\Rightarrow$ Factor Incomes.

How income is distributed to L and K

total labor income = $\frac{W}{P}\bar{L} = MPL \times \bar{L}$

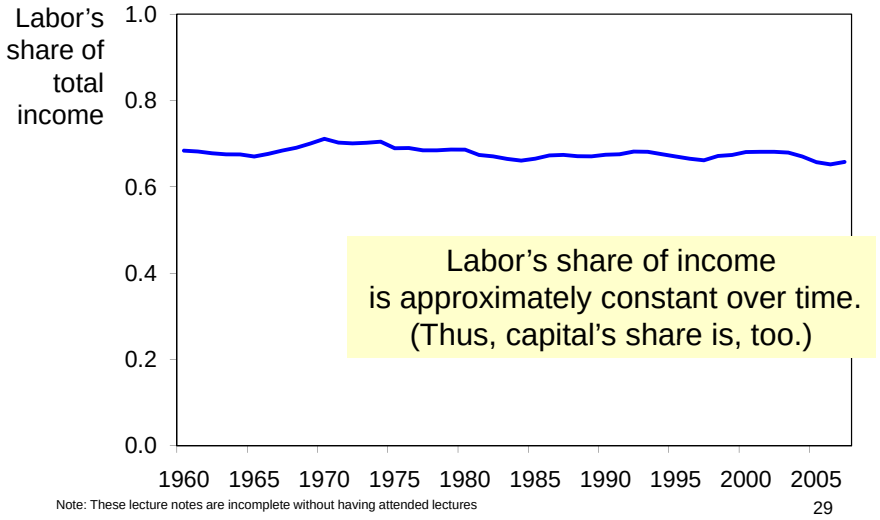
total capital income = $\frac{R}{P}\bar{K} = MPK \times \bar{K}$

If production function has constant returns to scale, then

$$\bar{Y} = \underbrace{MPL \times \bar{L}}_{\text{labor income}} + \underbrace{MPK \times \bar{K}}_{\text{capital income}}$$

national income

The ratio of labor income to total income in the U.S., 1960-2007



Putting It Together: An Example

- Competitive Firm maximizes Profits in the Long Run:

$$\max_{K,L} \Pi = PY - WL - RK \qquad (1)$$

subject to constraints that

i. Y, K, L satisfy the production function i.e.

$$Y = F(K,L) \qquad (2)$$

ii. and W, R are given

Optimality Conditions

- First Order Conditions (FOC):
  - With respect to capital: $\frac{\partial \Pi}{\partial K} = PF_K(K,L) - R = 0$ (3)
  - With respect to labor: $\frac{\partial \Pi}{\partial L} = PF_L(K,L) - W = 0$ (4)
- Second Order Conditions (SOC):
  - Satisfied by the properties of F(K,L), i.e.:

$$\frac{\partial^2 \Pi}{\partial K^2} \equiv PF_{KK} < 0; \quad \frac{\partial^2 \Pi}{\partial L^2} \equiv PF_{LL} < 0;$$

Neoclassical Theory of Distribution (cont.)

- Define:
  - $F_K(K,L)$ = MPK - marginal product of capital
  - $F_L(K,L)$ = MPL - marginal product of labor
- Define "real prices":
  - $r = \frac{R}{P}$
  - $w = \frac{W}{P}$

Demand for Factors

- Factors are paid their marginal products
  - MPK = r i.e. $F_K(K,L) = r$
  - MPL = w i.e. $F_L(K,L) = w$
- These yield the demand for factors
  - $K^d = Lg(r)$ $g'(r) < 0$
  - $L^d = Kf(w)$ $f'(w) < 0$
- Question: For the Cobb Douglas Production Function, derive the factor demands
  - $Y = AK^{\alpha}L^{1-\alpha}$

Supply of Factors and Factor Prices

- Supply of factors are fixed: $\bar{K}, \bar{L}$
- Factor prices are obtained by equating factor supply to factor demand:
  - $K^d = \bar{K}$
  - $L^d = \bar{L}$
- Thus solving for equilibrium r and w:
  - $\bar{L}g(r) = \bar{K} \Rightarrow g(r) = \frac{\bar{K}}{\bar{L}}$
  - $\bar{K}f(w) = \bar{L} \Rightarrow f(w) = \frac{1}{\bar{K}/\bar{L}}$

Equilibrium Factor Prices

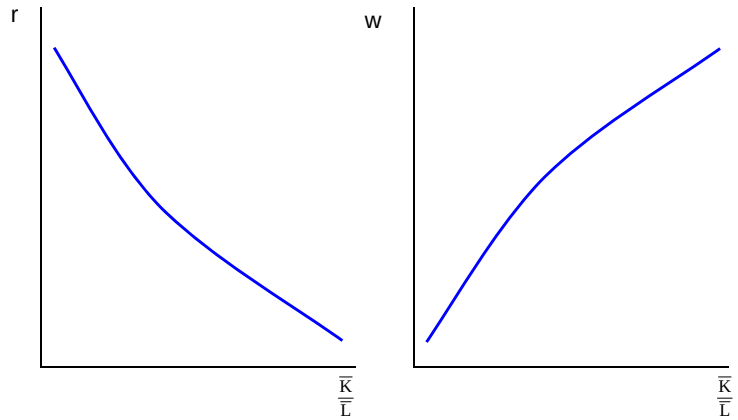


Illustration: Cobb Douglas Production Function

- For the Cobb Douglas Production Function:
  - $r = \alpha A\left(\frac{\bar{L}}{\bar{K}}\right)^{1-\alpha}$
  - $w = (1-\alpha)A\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha}$
- Question: Calculate the elasticity of r and w with the capital labor ratio.
  - For r:
  - For w:
- Note: These functions are constant elasticity functions

Total Factor Incomes

- Capital Income: rK
- Labor Income: wL

- Since r = F_K and w = F_L, sum of incomes at full employment is:
  - $\bar{K}F_K + \bar{L}F_L = \bar{Y}$ by Euler's Theorem

- Factor Shares in National Income:
  - Capital Share: $\frac{\bar{K}F_K}{\bar{Y}}$
  - Labor Share: $\frac{\bar{L}F_L}{\bar{Y}}$

Cobb Douglas Production Function Example (cont.)

- Marginal Product of Capital, $F_K = \alpha A K^{\alpha-1} L^{1-\alpha}$
- Multiply both sides by K: $KF_K = \alpha A K^{\alpha} L^{1-\alpha}$
  $= \alpha Y$
- Hence, share of capital = α
- Similarly, share of labor = 1- α
- Thus, the ratio $\frac{wL}{rK} = \frac{1-\alpha}{\alpha}$, which is a constant

Answers to Initial Questions…

- What determines Output in the Long Run?
  - The supply of factors gives total inputs, the production function transforms them into output.

- What determines Incomes of Factors of Production?
  - Their Marginal Products.

- How is Output distributed?
  - By the demand for goods and services: Consumption - C, Investment - I, Government Purchases - G, and Net Exports – NX.

Consumption

- Recall, consumption is biggest component of expenditure, nearly claiming 2/3 of output.

- As before, from the Short Run, i.e. $C=C_0 + c(Y-T)$. In general, consumption function is derived from Utility Maximization.

- Recall, from Time Series Data, $C_0 \approx 0$, so let $C=c(Y-T)$
  MPC = $C'(Y-T) = c > 0$ and < 1
  APC = $C/Y < 1$

Investment

Several different kinds:

1. Fixed investment: additions to capital stock
2. Replacement Investment: to replace depreciating capital stock
3. Stocks, works in progress, inventories: additions to the stock of unsold goods held by firms and to half-finished goods.
4. Residential Investment: additions to housing stock

- In the long run, we assume there is only one kind of investment (we aggregate all kinds into one) :–
  - Additions to capital stock.
  - Depreciation will be added later.

What does Investment Depend On?

- Recall in short run, I(r).
- Basic intuition is same in long run.
- Consider a firm thinking about adding one machine to its capital stock:-
  - Additional machine increases output by: x units per year (no depreciation).
  - Price of output in base year is 1, but firm expects inflation π per year.

- Addition to nominal revenues = $x + (1+\pi)x + (1+\pi)^2x$ $\ldots+(1+\pi)^{n-1}x+\ldots$
- Firm can raise funds at cost i per year.
- Question: How much is firm prepared to "buy" the income stream x, $(1+\pi)x$, … given cost of funds i?
- Answer: Present Discounted Value of income stream:

$$PDV=\frac{x}{1+i}+\frac{x(1+\pi)}{(1+i)^2}+\frac{x(1+\pi)^2}{(1+i)^3}+\ldots$$

$$\therefore\ PDV=\frac{x}{i-\pi}=\frac{x}{r}\qquad (r\equiv i-\pi)$$

Firm's Investment Decision

- Firm buys machine if its cost does not exceed its PDV
- If machine costs \$y:
  - Firm buys if: $y \le \frac{x}{i-\pi}$
  - Firm does not buy if: $y \ge \frac{x}{i-\pi}$
- i.e. investment occurs if: $\frac{x}{y} \ge i-\pi$

Firm's Investment Decision (cont.)

- x/y is the marginal rate of return on the machine, what Keynes called the Marginal Efficiency of Capital.
- It is influenced by:-
  - the capital/labor ratio.
  - entrepreneurial expectations, or as Keynes called them "animal spirits". If "business confidence" is high, x/y is expected to be high.

Investment in the Long Run

- $r \equiv i - \pi$ is defined as the **real** rate of interest. Also referred to as the *real cost of capital*.
- $I = I\left(\frac{K}{L}, r, \text{expectations}\right) \quad ; \quad I'(r) < 0$

  (-) (-) (+)
- For the short run analysis, we focused on I(r)
- For long run analysis: $I = \Delta K = I(k, r)$

  where $k = \frac{K}{L}$, and $I_k < 0$, $I_r < 0$.

Government Spending

- Government spends money on:-
  - Consumption goods, e.g. guns, services, such as public administration.
  - Investment goods, e.g. roads, hospital, "infrastructure".
  - Transfers, e.g. payment to unemployed.
- Usually treat government spending as consumption spending $\therefore$ G assumed exogenous.
- Here we ignore transfers.

Twin Deficits Identity

- In equilibrium, since output is distributed amongst consumption, investment, government expenditure and net exports:

  $Y = C + I + G + NX(Y-T, Y_f, \varepsilon)$
- Disposable income Y-T is either saved or consumed:

  $\therefore Y - T = C + S$

$\Rightarrow C + S_p + T = C + I + G + NX$

or $S_p - I = G - T + NX$

(Private savings less Investment) = (Budget Deficit) + (Balance of Payments Surplus)

Long Run Theory of Interest

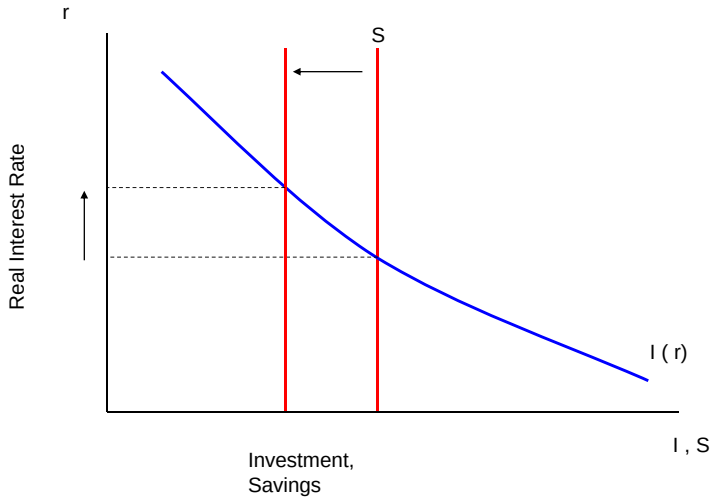
- $S_p - I = G - T + NX$ holds in both short and long run.

- Short run :–
  - monetary influences on r are important.

- Long run:-
  - Supply of K is fixed, or given in the long run.
  - But, for a given K and NX = 0:

$$I(r) = S_p(Y - T) + T - G$$
$$= national\ savings$$

Effect of Increase In Budget Deficit



Summary

1. Output in the long run is determined by factors of production
   - The supply of factors gives total inputs, the production function transforms them into output.

2. Incomes of factors of production are determined by their marginal products

3. Output is distributed amongst consumption, investment, government purchases and net exports in the long run

Summary

4. Treat Consumption as in the short run and focus on relationship between consumption and disposable income

5. Investment in the long run occurs if the marginal efficiency of capital exceeds the real cost of capital

6. Investment adds to the stock of capital

Summary

7. Government spending and net exports are as before in the short run

8. Twin deficits identity holds in the long run

9. The real interest rate is determined from the market for capital and equals the marginal product of capital (less the rate of depreciation).